ACOUSTIC REFLECTIONS AND TRANSMISSIONS BY THE ULTRASONIC METHOD

By

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An investigation carried out under the direction of Dr. R. W. Boyle, and presented to the Committee on Graduate Studies of the University of Alberta in partial requirement for the degree of Master of Science.

April 15, 1929.

INTRODUCTION

In the main, the physical study of acoustics is the study of a compressional wave in a three dimensional medium. Strictly speaking, however, it is not possible to observe directly the wave in the medium. Our present knowledge of a wave is based almost entirely on the observation of phenomena occurring at, or caused by, a discontinuity in the medium, and the behaviour of the wave in the medium is deduced from the facts so ascertained.

In illustration it is enough to point out that sound waves universally seem to be started at a discontinuity and detected by a discontinuity. The outstanding exception is the observation of the progress of a compressional wave by the change of refractive index, as in the well known spark experiments of Teepler, Wood and others. But the phenomena of reflection, refraction and diffraction are all produced by a discontinuity existing in an otherwise homogeneous medium.

A discontinuity may be caused by any one of a number of materials, each of which may possess different geometrical shapes. Hence it is essential that a systematic study be made of the effect of progressive variations in the material and in the shape of the material, if for no other reason than the facilitating of the study of the wave itself.

At first glance it would seem that the simplest case for both theoretical and experimental investigations would involve a plane wave incident on a plane interface between two media, each extending to infinity in opposite directions. A theory for this case has been developed by Poisson, Green and Rayleigh.[1]

In the case of ultrasonic waves, Boyle and Reid[2] made approximate measurements of the ratio of reflected to incident intensity for marble, limestone and steel reflectors of ultrasonic waves in water, but the precision of the experiments was not sufficient to constitute a verification of existing theory. A plane wave incident on a plane partition of finite thickness is simpler than the above case but beset with much greater theoretical difficulties. One out of many of these unsolved problems arose out of the above work by Boyle and Reid. They found that for a certain angle a large proportion of sound energy passed through a thin steel plate. Rayleigh has developed an expression for transmission by thin plates at angles of incidence greater than the critical angle, but the theoretical results are not in accord with Boyle and Reid's experiment.

The present investigation is divided into three parts, viz.

Part I. On energy reflections and transmissions at perpendicular incidence, this being a continuation of former work by Boyle, Lehman and Froman;

Part II. The main part of this report, on transmission by thin plates at oblique angles of incidence;

Part III. Ultrasonic transverse ~~trans~~vibration in bars and sheets.

PART I.

A related body of theory outlined by Rayleigh[3] has been verified. This outline indicated a mathematical procedure in the case of a plane faced partition of finite thickness, for angles of incidence less than the critical angle if a critical angle existed; and it found expressions for the ratio of reflected and transmitted energy, in terms of the respective densities and elastic constants of the two media, the thickness of the partition, and the angle of incidence.

Boyle and Rawlinson[4] expanded Rayleigh's condensed proof and arrived at some detailed conclusions submissible to experiment. Using the symbols employed by these latter workers, the ratio of transmitted to incident energy for any angle of incidence less than the critical angle, should one exist, is given by

$$\frac{E^2}{A^2} = \frac{4\operatorname{cosec}^2 \frac{2\pi \ell}{\lambda_1} \cos\theta_1}{4\cot^2 \frac{2\pi \ell \cos\theta_1}{\lambda_1} + \left(\frac{\rho V \cos\theta_1}{\rho_1 V_1 \cos\theta} + \frac{\rho_1 V_1 \cos\theta}{\rho V \cos\theta}\right)^2} \qquad \text{I.}$$

Where A^2 = Incident energy intensity

E^2 = Transmitted energy intensity

ℓ = Thickness of partition

λ_1 = Wavelength in the material of the partition

θ = Angle of incidence

θ_1 = Angle of refraction

ρ = Density of propagating medium

ρ_1 = Density of the material of the partition

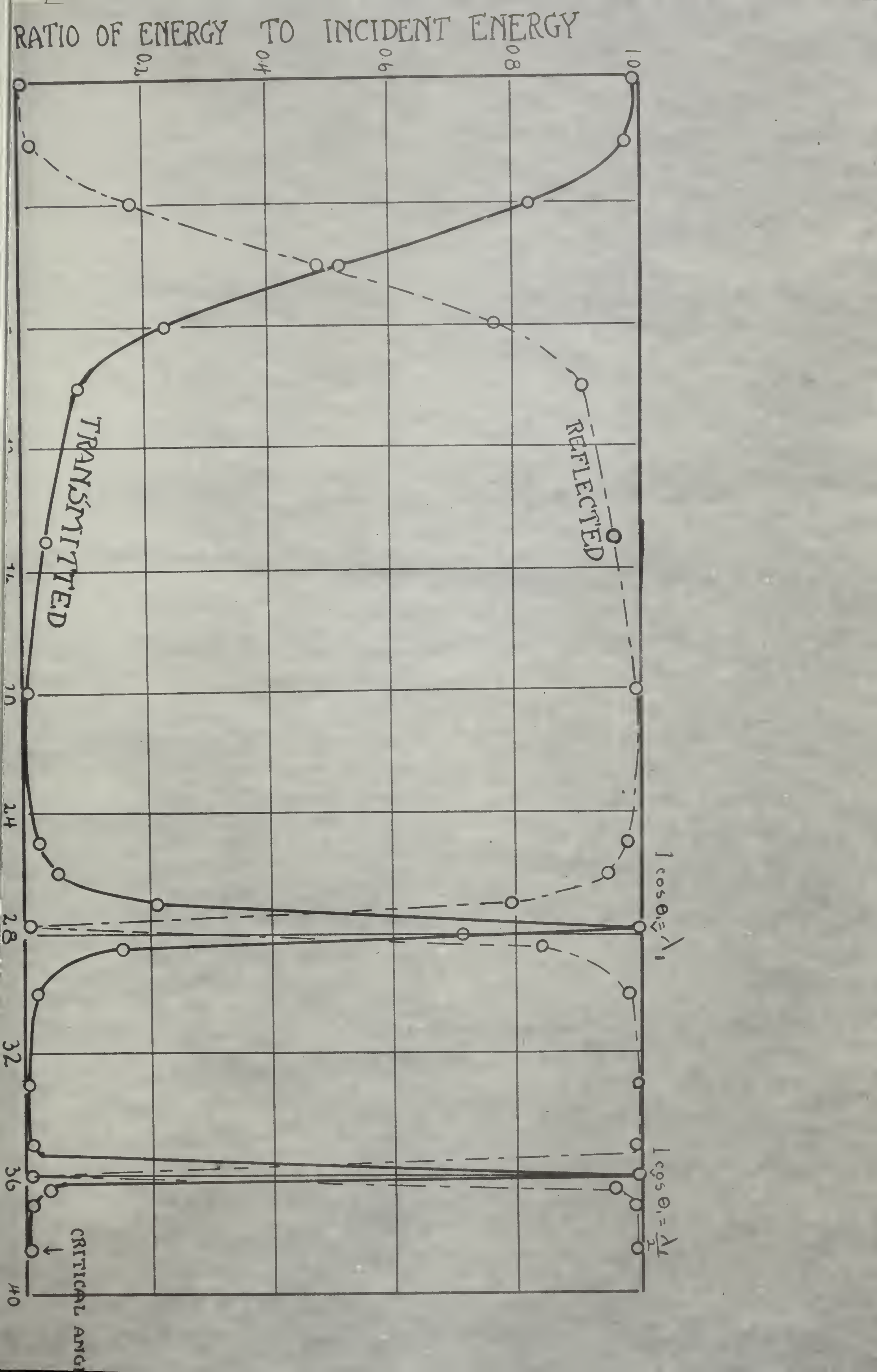
V = Velocity in the propagating medium

V_1 = Velocity in the partition

The ratio of reflected to incident energy is given by

$$\frac{B^2}{A^2} = \frac{\left(\frac{\rho V \cos\theta_1}{\rho_1 V_1 \cos\theta} - \frac{\rho_1 V_1 \cos\theta}{\rho V \cos\theta_1}\right)^2}{4\cot^2 \frac{2\pi \ell \cos\theta_1}{\lambda_1} + \left(\frac{\rho V \cos\theta_1}{\rho_1 V_1 \cos\theta} + \frac{\rho_1 V_1 \cos\theta}{\rho V \cos\theta_1}\right)^2} \qquad \text{II.}$$

where B^2 = Reflected energy intensity



RATIO OF ENERGY TO INCIDENT ENERGY
0.2
0.4
0.6
0.8
1.0
TRANSMITTED
REFLECTED
20
24
28
32
36
40
$\cos\theta_1 = \lambda_1$
$\cos\theta_1 = \frac{\lambda_1}{2}$

From (II) it may be seen that there is in general, minimum reflection and maximum transmission when $\frac{\cot 2\pi l \cos \theta_1}{\lambda_1} = \infty$ or when $l \cos \theta_1 = m\lambda_1$, where $m = 1, 2, 3,$ etc.

This may be illustrated by the theoretical curve Fig. I given as an example by Boyle and Rawlinson for the case of a plate of type-metal 3/2 λ thick, in water.

It should be noted that the theoretical curve must terminate at the critical angle, for <u>on the assumption made in the derivation of the expression, total reflection occurs at that angle of incidence and</u> $\frac{E^2}{A^2} = 0$, <u>for all greater incident angles</u>. By a modification of the above theory, Rayleigh showed that for a plate which is thin in comparison with a wave length, some energy should be transmitted even for angles of incidence greater than the critical angle. This theory will be discussed later and will be shown to be inadequate to explain the results of the present investigation in Part II.

For normal incidence, $\theta_1 = \theta = 0$ and (1) reduces to

$$\frac{E^2}{A^2} = \frac{4 \operatorname{cosec}^2 \frac{2\pi l}{\lambda_1}}{4 \cot^2 \frac{2\pi l}{\lambda_1} + \left(\frac{\rho V}{\rho_1 V_1} + \frac{\rho_1 V_1}{\rho V}\right)^2} \qquad \text{III}$$

while (II) reduces to

$$\frac{B^2}{A^2} = \frac{\left(\frac{\rho V}{\rho_1 V_1} - \frac{\rho_1 V_1}{\rho V}\right)^2}{4 \cot^2 \frac{2\pi l}{\lambda_1} + \left(\frac{\rho V}{\rho_1 V_1} + \frac{\rho_1 V_1}{\rho V}\right)} \qquad \text{IV}$$

It should be noted that it follows from (IV) that the reflected energy is minimum and the transmitted energy is maximum for $l = m\frac{\lambda}{2}$ where m = 1, 2, 3, etc.

It is apparent from (III) that there are two ways of changing E^2/A^2 for a given material. One is to change l by using a different thickness, Method A; the other is to keep the thickness constant and vary λ by changing the frequency Method B. The

most practical method of testing the theory by means of the experimental facilities available in this laboratory is a combination of the two modes of procedure. The application of the variable thickness method (A) is limited by virtue of the fact that it would be necessary to make a series of plates of gradually increasing thickness, each one larger than the one before by 1 % in order to obtain 1 % accuracy. This is tedious and hardly practicable. On the other hand, the variable frequency method (B) is limited by the restricted range of frequencies available with the ordinary type of resonant ultrasonic generator which must be used as sources of the waves. Hence the following was the general procedure here adopted.

The ratio E^2/A^2 was determined for a series of plates of gradually increasing thickness keeping the frequency constant. The plate showing maximum transmission was then chosen for further experiment. In general, this plate would not be an exact multiple of $\lambda/2$ in thickness; but, from the sharpness of the maximum peak of the transmission curve, it could be discovered that the plate very nearly fulfilled this condition. Using this plate, a slight variation in frequency in the right direction should result in a further increase in the transmission ratio. The precise experimental method of finding the frequency at which this rise occurs will be described later.

The theoretical results quoted above are of much practical interest and importance. On the experimental side there is a great difficulty in carrying out quantitative work on the problem at audible frequencies, and, until recent years, the lack of suitable sources and receivers prevented satisfactory experimental work at high frequencies. Thus no experimental verification of the theory by Rayleigh and his

contemporaries was possible or attempted. To note only one of the numerous difficulties attendant upon the use of low frequencies, consider the dimensions of the plate required when using the frequency of midd middle C, 256$\sim$/sec. In order to determine the transmission for a partition of type-metal, say 1.8λ in thickness, i.e. more than three half wave-lengths, a plate 16 metres thick would be required; and such a plate ought to be in area at least 250 metres square. But, using an ultrasonic frequency of 308,000$\sim$/sec., a plate of the same material as thin as 1.4 cms. and 23 cms. square would serve quite well. Hence it may readily be seen that the development of the ultrasonic beam transmitter and the convenient use of the torsion pendulum by means of which either relative or absolute measurements of ultrasonic intensity could be made, opened up a new region of experiment in acoustics and wave motion in general. A class of experiments involving apparatus of the order of a wavelength, feasible neither in sound, owing to the great length of audible sound waves, nor in light, owing to the extreme shortness of light waves, could now be readily performed using ultrasonic waves of the order of a millimeter to several centimeters in length.

EXPERIMENTAL

Method A.

/ In 1922, Boyle and Lehmann[6] tested the theory outlined above for normal incidence, varying the thickness of the partition. The ultrasonic method employed was very ~~ingenious and~~ simple. By making the reflecting partition in the form of the vane of a torsion pendulum the partition itself was made to indicate the ratio of reflected to incident energy, as the resultant radiation pressure on such an obstacle is equal to the difference in energy density on its two sides. To test the theory it was simply necessary to suspend pendula vanes of various thicknesses, but of the same area, in an ultrasonic beam propagated in water, and plot a curve of pendulum readings against ℓ/λ.

It should be noted that in the case of a metal pendulum, the deflection is proportional to the difference in energy density on each side of the reflecting vane. Expressed mathematically, the deflection

$$d = k(A^2 + B^2 - E^2), \text{ where K is a constant}$$

$$\text{But } A^2 = B^2 + E^2$$

Substituting,

$$d = K(B^2 + E^2 + B^2 - E^2) = 2kB^2$$

Hence the deflection is proportional to the reflected energy. But, in the case of the deflection of the pendulum by the energy passing through the plate, $d = kE^2$, so the curves obtained in the two cases by plotting d against ℓ/λ should be complementary. That such is the case may be seen by referring to Figs. III and IV, (following Page 11), which were obtained experimentally in the course of this investigation.

In 1923 Boyle and Lehmann measured the relative ~~actual~~ intensity of the energy of an ultrasonic beam transmitted normally through metal and other

plates of thicknesses in various ratios to a wave-length. These hitherto unpublished results are related to the present investigation, and as permission has been kindly granted by the authors, they will be included in this report.

A light wooden framework, annular in shape, supported the experimental plate in a large tank of water through which the ultrasonic beam was projected. This tank, which has already been described[2], was 15 ft. long by 5 ft. wide, by 3 ft. 6in. deep. The diameter of the circular aperture of the framework was just less than 12 inches, and when open the ultrasonic beam could pass centrally through the aperture at right angles to its plane. The aperture could be closed by the experimental plate, which was 12 inches square, using the plate as a shutter, and taking care to keep it accurately perpendicular to the beam.

Observations of energy intensity were made by means of an appropriately designed torsion pendulum, suspended on the axis of the beam not far from the framework on the side remote from the ultrasonic transmitter. When the aperture was open the reading of the torsion pendulum gave a measurement of the incident energy within the required limit of accuracy; when it was closed by the plate all other conditions being the same, the reading gave a measurement of the energy transmitted through it.

Hence the ratio of transmitted to incident energy at various frequencies, could easily be ascertained. There were some reflections of energy, of course, from the wooden framework supporting the plate, and these would interfere somewhat with the incident beam. For this reason the experimental arrangement is not considered to be as good or as accurate as the one used presently, viz. suspension of

the plate in the path of the beam by means of thin wires hanging from top supports.

The procedure of an experiment was to measure, by means of the torsion pendulum, the energy emission through the circular aperture, both with and without the plate shutter, over a range of frequencies including that particular frequency for which the plate was one half wave length thick. Corresponding curves of energy transmission through the plate were plotted on a frequency base. From the curves the ratio of transmitted to incident energy could be found for any frequency within the range of the experiment. If, as according to the theory, there is maximum transmission when the plate is exactly one half wave length thick, this ratio should be markedly increased at the exact frequency which makes the shutter a half-wave plate. Experimentally this was shown to be the case. As the frequency was increased and the appropriate half wave length was approached the transmitted to incident energy ratio rose, and fell again where this appropriate frequency was exceeded. With plates of metals and of glass, the rise and fall in the ratio was quite sharp, but less sharp in materials like wood and stone. Most of Boyle and Lehmann's experiments were performed with steel plates, but there were a few experiments, using other materials, viz. lead, glass, marble and wood. Their results are included here in Table I. The ultrasonic generator used in their experiments was one of the double steel plate (2 quarter wave plates) type, with 6 inch diameter of radiating face. It was set up about 164 cms. from the measuring pendulum, and projected its emitted beam horizontally in the tank. The usual high frequency voltage imposed on the generator during the experiments was about 2000 volts. The torsion pendulum used was of the single vane type 1.5 cms. in diameter, with phosphor bronze strip suspension 0.0025 inch wide and about 80 cms. long.

TABLE I

Material	Temperature	Thickness of Plate	Frequency for Maximum Transmission	Wave Length	Vel. of Sound in Material
Steel	14.7° C.	2.15 cms.	132000 ~/sec.	4.30 cms.	5.68x 10^5 cms/sec.
Steel	13.2	2.15	263000 *	4.30	5.65
Steel	13.9	2.15	129900	4.30	5.59
Steel	13.2	1.55	185000	3.10	5.73
Steel	13.2	1.55	359000 *	3.10	5.56
Lead	11.0	0.814	140700	1.63	2.29
Lead	12.3	0.814	140000	1.63	2.28
Glass	9.7	0.992	273000	1.98	5.42
Marble	9.7	2.56	64000	5.12	3.28
Maple Wood (along fibre)	15.3	2.66	79500	5.32	4.23

* At these frequencies there were second transmission maxima, consequently the plate must then have been two half wave lengths thick.

It is of interest to remark that it was observed in Boyle and Lehmann's research on some of the steel plates that, when the plates were in the condition of exact resonance, (i.e. at their half-wave-length thickness) with the frequency of the impinging waves there was a suggestion of an increased intensity of the energy field along the axis of the beam at some distance from the plate. This was not impossible, since the plate was then acting as a tuned resonator in an imposed sound field. It is hoped to undertake a special research later to investigate this point specifically.

Method B.

In 1926-27 Boyle and Froman applied the change of frequency method (B) to partitions of lead, duraluminum, and paraffin. Later in 1927, Boyle and Sproule[9] applied both the pendulum and plate method to a test of the theory, using type metal as experimental material. Type metal was chosen because the "mass of a wave length" $(\rho\lambda)$ in this material was less than in most materials readily available, and therefore the experiments with it were speedier and less irksome. The pendula vanes were made 2.5 cms. in diameter, and from 0.228 cms. to 0.518 cms. thick. These pendula were suspended in the ultrasonic beam on the axis by a phosphor bronze wire 82 cms. long and 0.0025 in. in diameter. Two series of observations were taken, one at a frequency of 306,500 cycles/sec., the other at 529,500 cycles/sec.

A pendulum 2.5 cms. in diameter is not a very close approximation to a partition of infinite extent as postulated by theory. However, results obtained by placing plates 23 cms. square in the path of

the beam and measuring the intensity of the transmitted beam by means of an air vane single pendulum, indicated that

(a) the condition of a partition of infinite extent as postulated in the theory is satisfied as well by the pendulum vane as by the comparatively large plates:

(b) both cases afford substantiation of former results and verification of the theory for normal incidence.

Method A.

The plates used in method A ranged in thickness from 2.55 cms. to 0.197 cms., and were suspended normally to the beam by wires hanging from top supports, at a distance of approximately 1.5 meters from the transmitter. An air vane pendulum, described elsewhere[10], 1.5 cms. in diameter, was suspended on the axis of the beam about 5 cms. from the type metal plate on the side of it remote from the transmitter, by a phosphor bronze strip 0.0015 in. thick and 67 cms. long. As in the case of Method B, series of observations were taken at two different frequencies, viz. 306,500 cycles/sec. and 529,500 cycles/sec.

The proceeding indicated by the theory for the case of normal incidence was followed in both Method A and Method B experiments. In references to the experimental results it must be kept in mind that whereas an increase in transmission is indicated by a rise of deflection in the case of Method B, it is indicated by a decrease in deflection in the case of Method A.

The results shown in Tables II, III and IV were obtained as follows: The frequency and intensity of the beam were

TABLE II. (Method A)

Pendula of varying thickness - constant frequency .306500~/sec.

V_1 taken as 2.37 x 10^5 cms./sec. and calculated from $V_1 = f\lambda_1$

ℓ	B^2	A^2	B^2/A^2
.228	40.0	40.0	1.00
.314	39.0	"	.975
.354	33.5	"	.84
.369	23.0	"	.575
.387	19.5	"	.49
.395	11.0	"	.275
.424	27.5	"	.69
.449	37.5	"	.94
.518	39.0	"	.975
.611	39.5	"	.986

TABLE III. (Method A)

Pendula of varying thickness - constant frequency 529500 ~/sec.

V_1 taken as 2.37×10^5 cms. sec. and calculated from $V_1 = f \lambda_1$

l	B^2	A^2	B^2/A^2	l/λ_1
.131	38.7	38.7	1.0000	.252
.197	26.5	38.7	.940	.440
.211	26.3	38.7	.680	.470
.221	18.2	38.7	.470	.493
.228	13.0	38.7	.336	.508
.233	18.0	38.7	.405	.520
.239	27.1	38.7	.698	.533
.251	34.3	38.7	.890	.573

TABLE IV (a) (Method A)

Plates of varying thickness - constant frequency 308000 ~/sec.

V_1 taken as 2.37 x 10^5 cms./sec. and calsulated from $V_1 = f\lambda_1$

ℓ	E^2	A^2	E^2/A^2	ℓ/λ_1
.199	2	200	.010	.258
.3095	7	198	.035	.401
.4034	131	198	.660	.522
1.0940	10	198	.050	1.420
1.1980	34.5	192	.180	1.550
1.4070	3.5	198	.018	1.830

TABLE IV (b)

As above but frequency at 306500 /sec.

ℓ	E^2	A^2	E^2/A^2	ℓ/λ_1
.600	.5	196	.0025	.780
.646	4.3	196	.022	.837
.677	7.0	196	.036	.876
.727	10.5	196	.084	.943
.782	140.0.	196	.714	1.010
.801	44.5	196	.230	1.035
.902	1.9	196	.009	1.170
1.002	1.4	196	.007	1.300

FIG. II

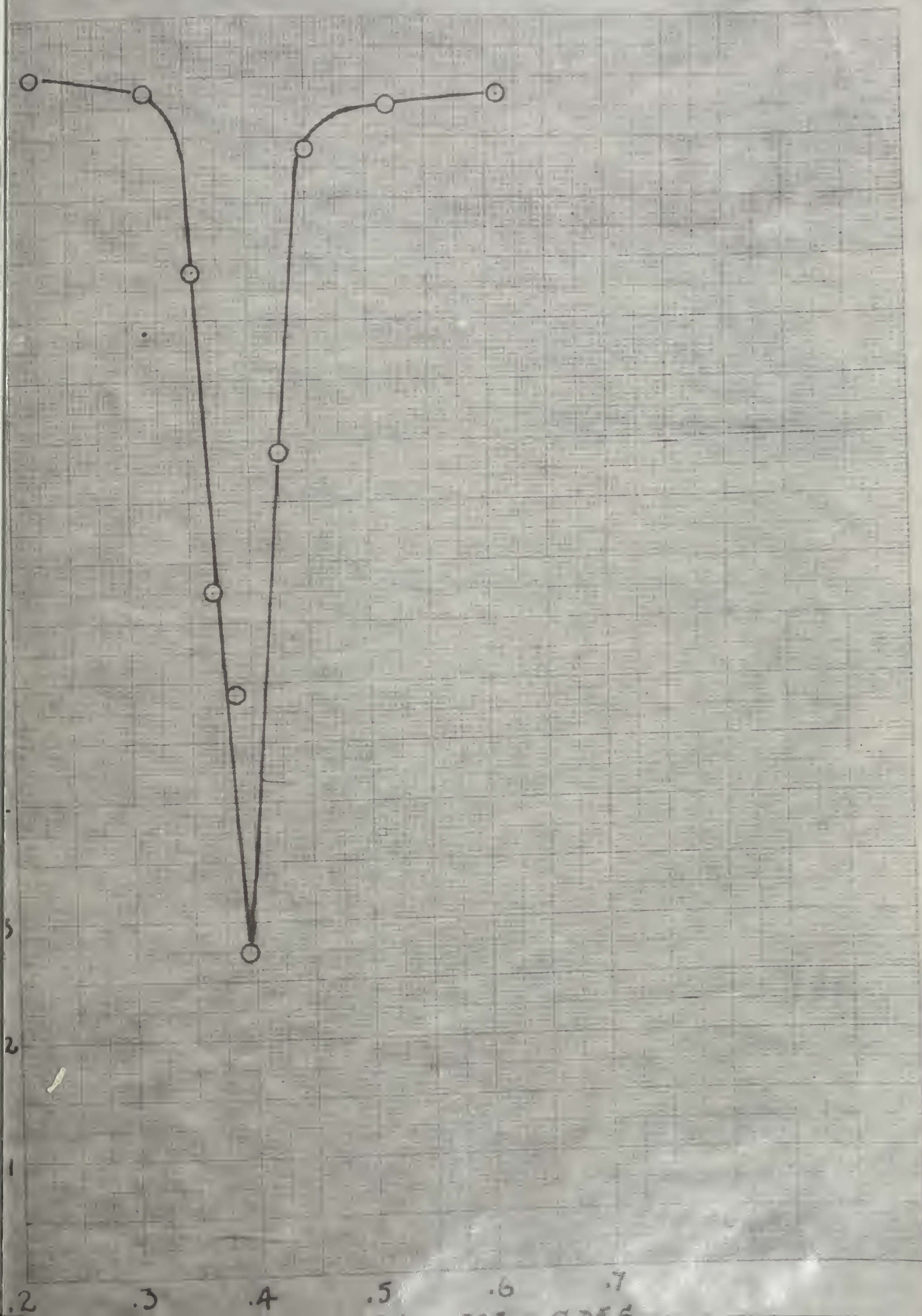
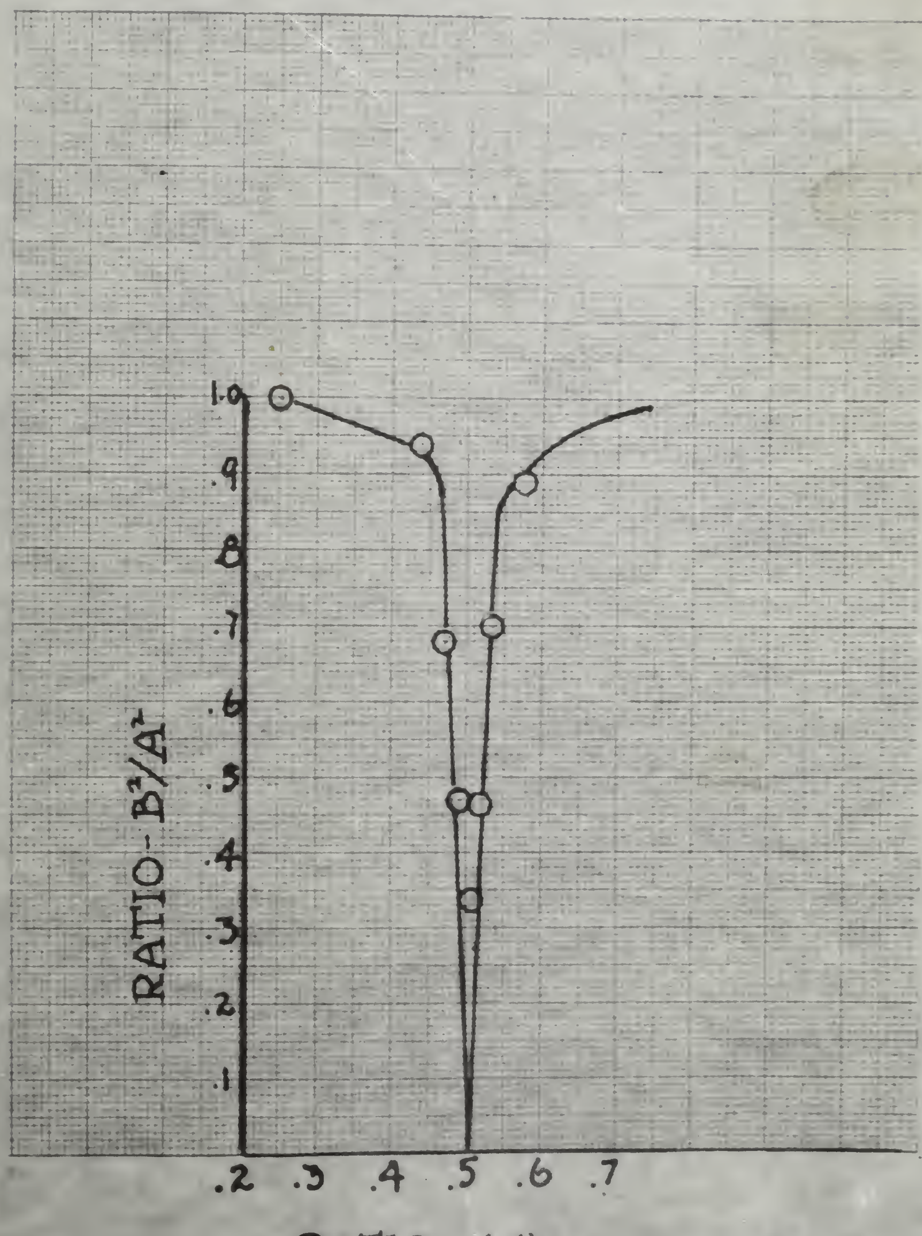


FIG. III



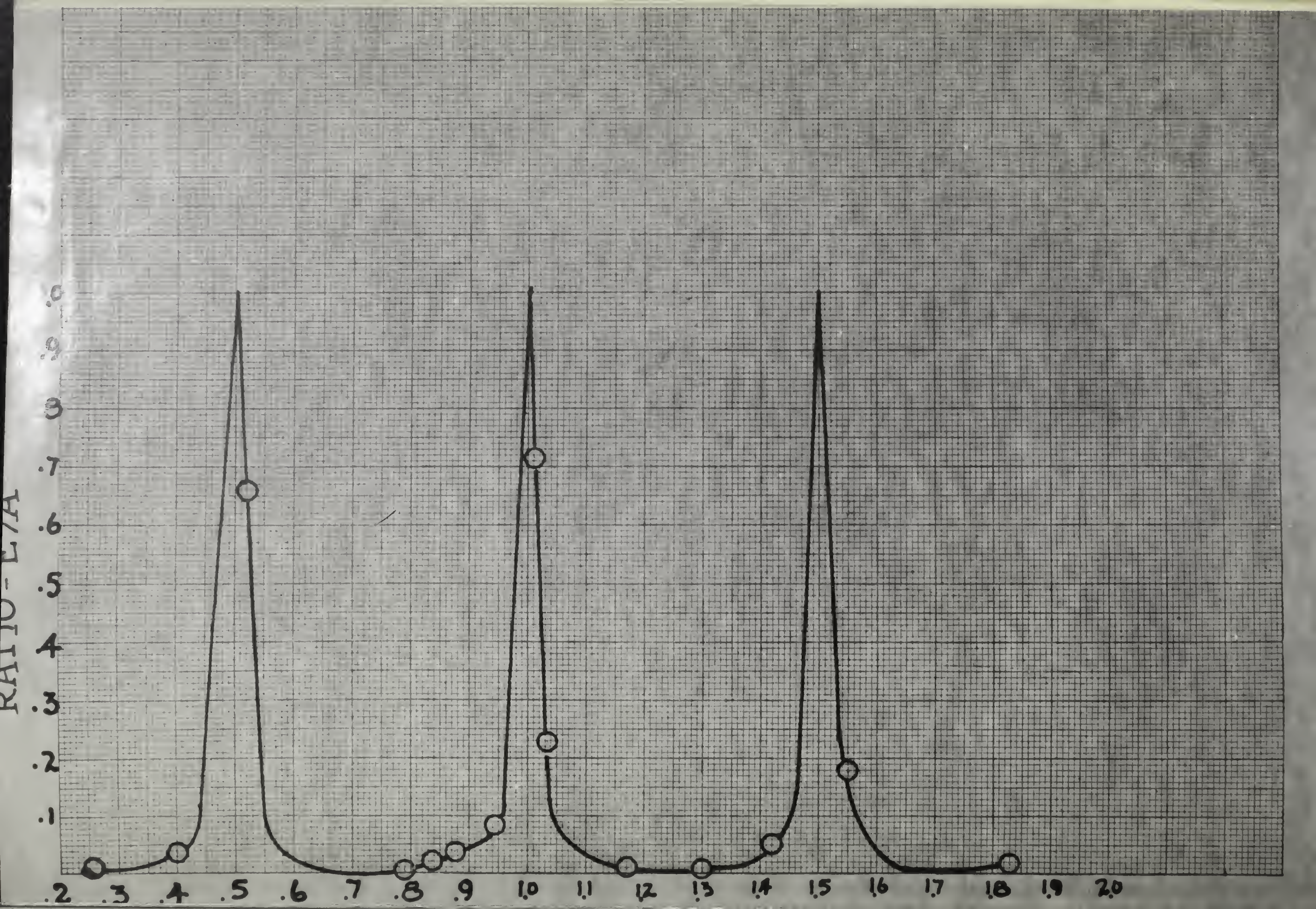
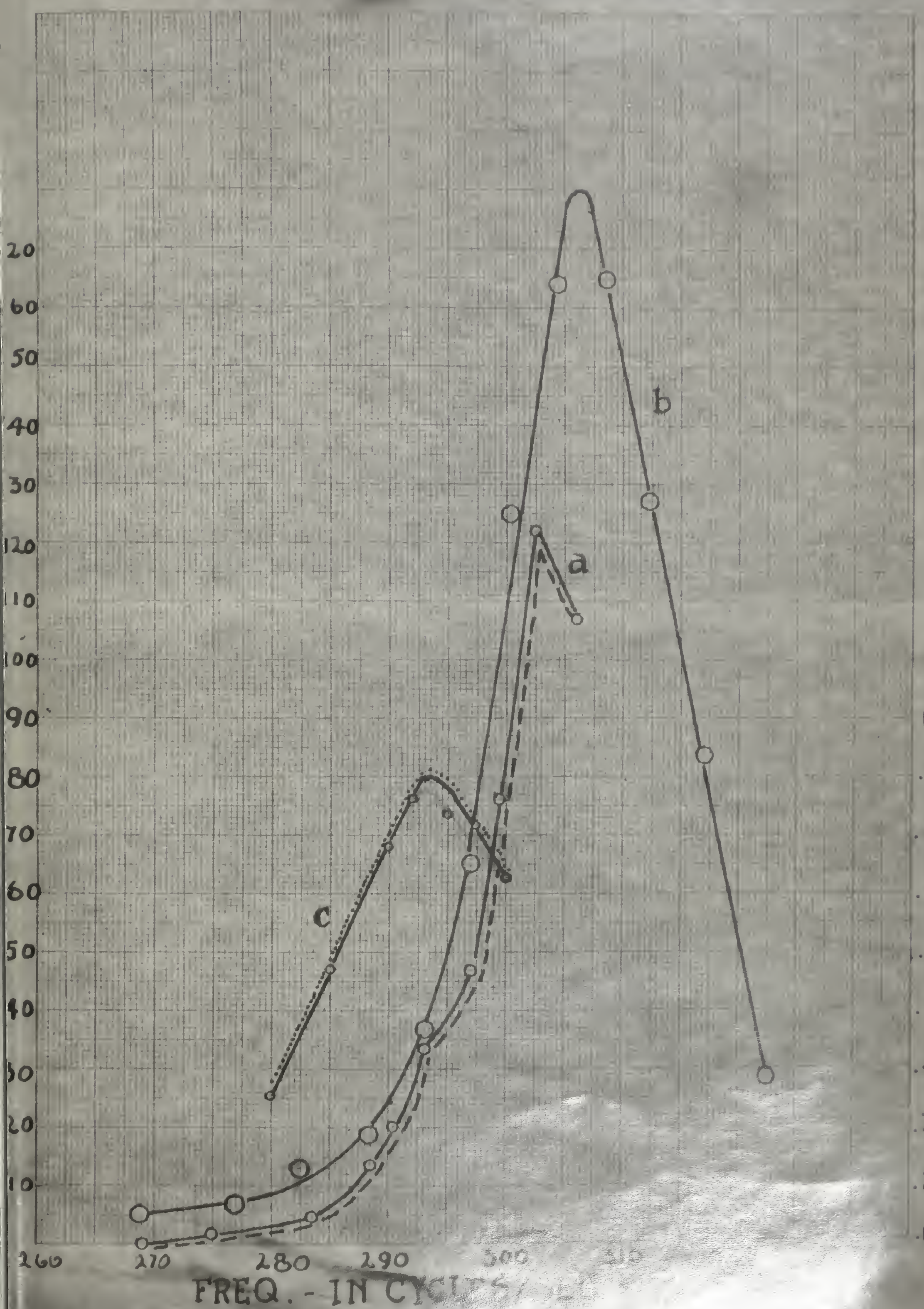

RATIO - E/A
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
.2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0

FIG. V



20
60
50
40
30
120
110
100
90
80
70
60
50
40
30
20
10
260
270
280
290
300
310
FREQ. - IN CYCLES/SEC
a
b
c
RATIO E°/A° (FOR CURVE C)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

in the beam the ultrasonic frequency was varied by small intervals and corresponding deflections noted and plotted. Curve (a) Fig. V was obtained, representing kE^2 against frequency (f). The plate was removed and another series of readings taken in the same manner. Curve (b) was plotted, showing kA^2 against f. From these two curves a third curve (c) was obtained representing E^2/A^2 against f. The peak of this curve occurred at frequency of 293600 ~/sec., at which frequency the plate must have been an exact $\lambda/2$. From this V was calculated as 2.37×10^5 cms./sec.

In a similar manner velocity determinations were made for the plates (Method A) and pendula (Method B) represented in Table V and V(a). The results for the pendula are not as consistent as the results for the plates but the mean values of velocity are in good agreement. Perhaps there should be slightly different velocities in the pendula vanes, for the method of casting the pendula and their comparatively small size made for rapid cooling, hence internal strains due to imperfect annealing could be set up in the vanes with a consequent slight change in their elastic qualities. This was not so apt to be the case for the plates.

Experiment at a Lower Frequency.

Using the velocity determinations already obtained the thicknesses of plate were calculated to correspond to $\lambda/4$ and $\lambda/2$ for a much lower frequency, viz. 45,600 ~/sec. and the plates constructed. The variable frequency method (B) could not be applied here for at this

TABLE V (Method A)

Type metal plate 0.403 cms. thick - varying frequency.

Plate Intervening See curve (a) Fig. V. Frequency	E^2	Without Plate See curve (b) Fig. V. Frequency	A^2
283500	5.0	277000	7.1
288500	14.0	285000	13.0
290500	20.0	288000	18.5
293000	34.0	293200	37.0
297000	47.0	297000	65.0
299500	76.0	300300	125.0

TABLE V (a) (Method B)

Pendulum results by varying frequency

Pendulum Thickness	f for $l = \frac{\lambda}{2}$	V_1
.387 cms.	303800 ~/sec.	2.35×10^5 cms./sec.
.395	304500	2.45 " "
.221	528000	2.33 " "
.228	529000	2.41 " "

Mean V_1 = 2.37

Plate Thickness	f for $l = m\lambda_1/2$	m	V_1
.4034 cms.	393600 ~/sec.	1	2.35×10^5 cms.sec.
.8011	296000	2	2.37 "
.672	527500	3	2.36 "
1.198	296000	3	2.37 "

Mean V_1 = 2.36

TABLE V (Cont.)

See curve (c) Fig. V.

Frequency	E^2	A^2	E^2/A^2
280000	3.0	8.5	.353
285000	7.5	13.0	.575
290000	18.0	23.0	.780
292000	28.5	33.0	.865
295000	41.0	49.0	.837
297500	59.0	72.0	.820
300000	84.0	115.0	.730

lower frequency the broadness of the beam allowed part of the energy to be reflected from the side of the tank, thereby forming a stationary wave system, as in the case of Young's mirror in optics. This made the pendulum deflection ~~exceedingly~~ very erratic. However, with the $\lambda/4$ plate in position not enough energy was transmitted through it to give any appreciable deflection of the indicating pendulum while the $\lambda/2$ plate allowed nearly 50 % of the energy to pass, so that this thickness of plate could not have been far from the actual $\lambda_1/2$ for that frequency, or the velocity in that plate must have been very nearly 2.37×10^5 cms./sec., since this was the velocity used in calculating $\lambda_1/2$.

The results indicate that in type metal there is no appreciable change in the velocity of a compressional wave over a frequency range of 45000 to 530000~/sec. or almost 4 octaves.

PART II.

Transmission of Wave Energy through Partitions at Oblique Angles of Incidence.

At this point of the investigation it was considered that the theory for normal incidence had been tested sufficiently, and the work on oblique angles of incidence was begun. Such work may involve the phenomenon of total reflection: in consequence some discussion of a mathematical analysis of this case as given by Rayleigh[1(a)] is included here.

To quote, (Sec. 270 - Vol. II - Theory of Sound) "Let us suppose that the medium is uniform above and below a certain plane ($x = 0$), but that in crossing that plane there is an abrupt variation in the mechanical properties on which the propagation of sound depends - namely the compressibility and the density. On the upper side of the plane (which for distinctness of conception we may suppose horizontal) a train of plane waves advances so as to meet it more or less obliquely; the problem is to determine the (refracted) wave which is propagated onwards within the second medium, and also that thrown back into the first medium, or reflected."

For angles of incidence equal to or greater than the critical angle θ_c given by $\sin\theta_c = \frac{V}{V_1}$, the incident, reflected and refracted waves may be expressed as follows:

Incident wave

$$\phi = \cos(ax + by + ct) \qquad \text{V.}$$

Reflected wave

$$\phi = \cos(-ax + by + ct + 2\epsilon) \qquad \text{VI.}$$

Refracted wave

$$\phi = \frac{2}{\left(\frac{\rho_1^2}{\rho^2} + \frac{a_1'^2}{a^2}\right)^{\frac{1}{2}}} e^{a_1' x} \cos(by + ct + \epsilon) \qquad \text{VII.}$$

From (V) and (VI) it can be seen that the incident and reflected waves differ only in phase and so reflection must be total. Yet the analysis shows that a disturbance does exist in the second medium. However, this apparent paradox may be explained by pointing out that the disturbance in the second medium is not a wave at all, although it is periodic, but dies out within a few wavelengths of the surface of separation. Analytically the optical case is similar. The corresponding equations may be found in Wood's Physical Optics, p. 273.

That such a disturbance does exist in the second medium may be shown by an optical experiment which dates back to the time of Newton and Fresnel. If a convex surface of glass of large radius of curvature is brought into contact with a plane glass surface at which total reflection is taking place the light will be found to enter the lens at the point of contact. That transmission should occur here is not in itself surprising as the point of contact is no longer a boundary, and so total reflection should not be expected to occur.

However, this point is surrounded by a ring which transmits light of a reddish tinge, at the same time reflecting light of a bluish tinge. The glass surfaces are not in contact here, but the air film is too thin for total reflection to take place. Transmission through the thicker portions of the ring will occur more readily for red light, as the thickness necessary to reflect is measured in terms of the wavelength. Another, and simpler, demonstration of the existence of the above mentioned disturbance in the second medium may be obtained by lightly smoking one surface of a right-angled prism with a flame. On sending a strong beam of light into the prism the smoked patch only will be illuminated, as total reflection occurs at all other points on the surface.

An indication that the same sort of thing is true for a sound wave is the experiment on thin steel plates, by Boyle and Reid[2].

Rayleigh has given expressions for incident and transmitted waves, incident on a thin plate at an angle greater than the critical angle.

Incident wave, $\phi = \cos(ax + by + ct)$ VIII

Transmitted wave, $\phi = \dfrac{2\sin(ax + by + ct + a\ell + \epsilon)}{\{4\cosh^2 a'\ell + (\alpha'^{-1} - \alpha')^2 \sinh^2 a'\ell\}^{\frac{1}{2}}}$ IX

where a, a', c, α' are all constants for a given material at a fixed angle of incidence. For their physical significance, see Rayleigh "Theory of Sound", Sect. 270-271, Vol. II.

ϵ is given by $\cot \epsilon = \frac{1}{2}(\alpha'^{-1} - \alpha') \tanh a'\ell$ X

From (IX) it may be seen that if $\ell = 0$, $\cosh^2 a'\ell = 1$, and $\sinh^2 a'\ell = 0$. Also in (X) $\tanh a'\ell = 0$, as $\epsilon = \frac{\pi}{2}, 3\frac{\pi}{2}$ etc. and therefore the transmitted wave becomes

$$\phi = \cos(ax + by + ct)$$

which is identical with the incident wave, as is to be expected, for $\ell = 0$ means no plate is present.

Supposing the angle of incidence to remain constant, it may be seen that as ℓ increases both $\sinh a'\ell$ and $\cosh a'\ell$ increase, therefore, from (IX) the intensity of the transmitted wave must decrease steadily and ~~roughly~~ approximately as $e^{-a'\ell}$. That this is not the case will be seen in the discussion which follows. But, as shown by the results obtained in this investigation and also by former results of Boyle and Reid[2], it is true that there is very significant energy transmission through plates which are thin in comparison with a wavelength at all angles of incidence, even for angles of incidence greater than the critical angle.

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

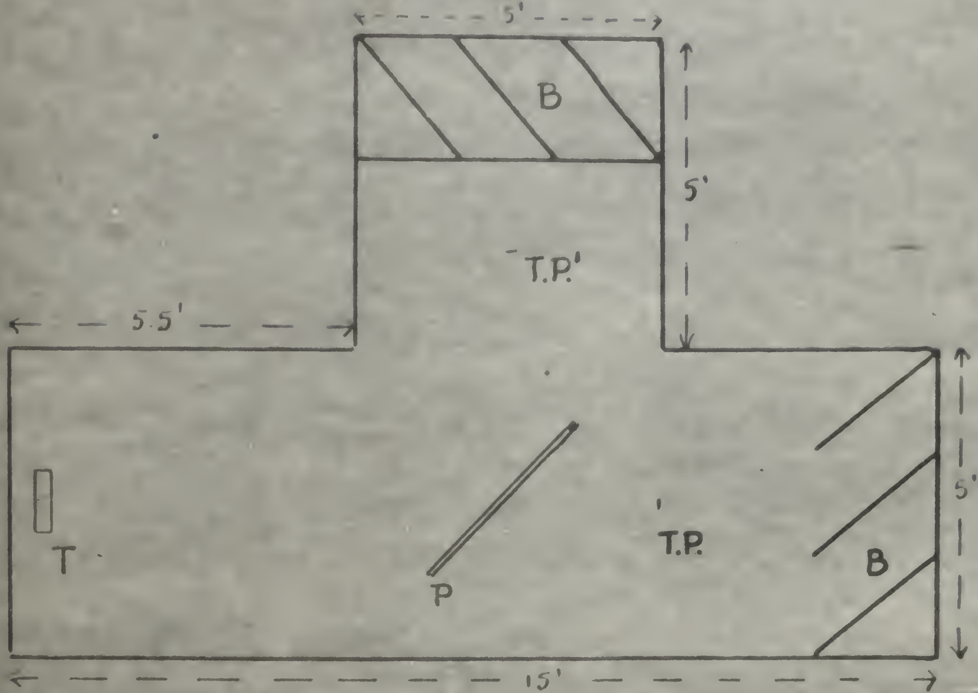
[illegible]

$$[illegible]$$

[illegible]

[illegible]

FIG. V-A



T- TRANSMITTER
P- PLATE
T.P.- TORSION PENDULUM
B- BAFFLE-BOARD

TANK FOR ULTRASONIC EXPERIMENTS

EXPERIMENTAL PROCEDURE.

The practical considerations of availability and cost led to the choice of glass as the reflecting material most suitable for the experiments. Due to the oblique angle of incidence, it was necessary to use larger reflecting sheets than for the case of normal incidence. The plates were cut 40" x 36" and ranged in thickness from 0.055 cms. to 0.88 cms. No facilities for ~~getting~~ procuring intermediate thicknesses were available, so stock sizes of window glass and plate glass were employed. The air vane pendulum used to measure the intensity of the beam was 2 cms. in diameter, and was suspended by a phosphor bronze wire 0.0025 cms. in diameter and 60 cms. long. The arrangement of the apparatus is shown in Fig. V-A.

The tank was about 3.5 ft. deep and the transmitter (T) and the torsion pendulum (TP) were suspended so that their centres were about 1.75 ft. from the top of the tank. The plate (P) was suspended with the long edge (40") horizontal. Two $\frac{1}{4}$-inch holes were drilled $1\frac{1}{2}$ inches from this edge, 10 inches from each end. The plates were hung by $\frac{1}{4}$ inch rope through these holes from a long 3-inch plank which passed from one side of the tank to the other. The angle of incidence was varied by sliding the plank along the edge of the tank. The position of the transmitter was adjusted so that the pendulum was on the axis of the emitted ultrasonic beam, as evidenced by maximum deflection of the pendulum. The baffle boards (B) were set so as to scatter the beam and prevent troublesome reflections at and from the ends of the tank. The first series of experiments was conducted using a frequency of 157,000$\sim$/sec. and a H.F. voltage on the transmitter of about 900 V.

The average of various quoted values for the velocity of

sound in glass is about 5.8×10^5 cms./sec. For water, $v = 1.48 \times 10^5$ cms./sec. Hence, from $\sin \theta_c = V/V_1$, the critical angle (a) for water to glass should be about 15°. However, the thinnest plate was only 0.055 in thickness, and with the above work by Boyle and Reid in mind, it was ~~exceedingly~~ very questionable as to whether total reflection would occur for any angles of incidence, even those greater than the critical angle.

To test the point the thinnest plate was suspended at an angle of 45° to the beam. Total reflection was easily disproved, as over 50 % of the incident energy passed through the plate, although the angle was so much greater than the critical angle.

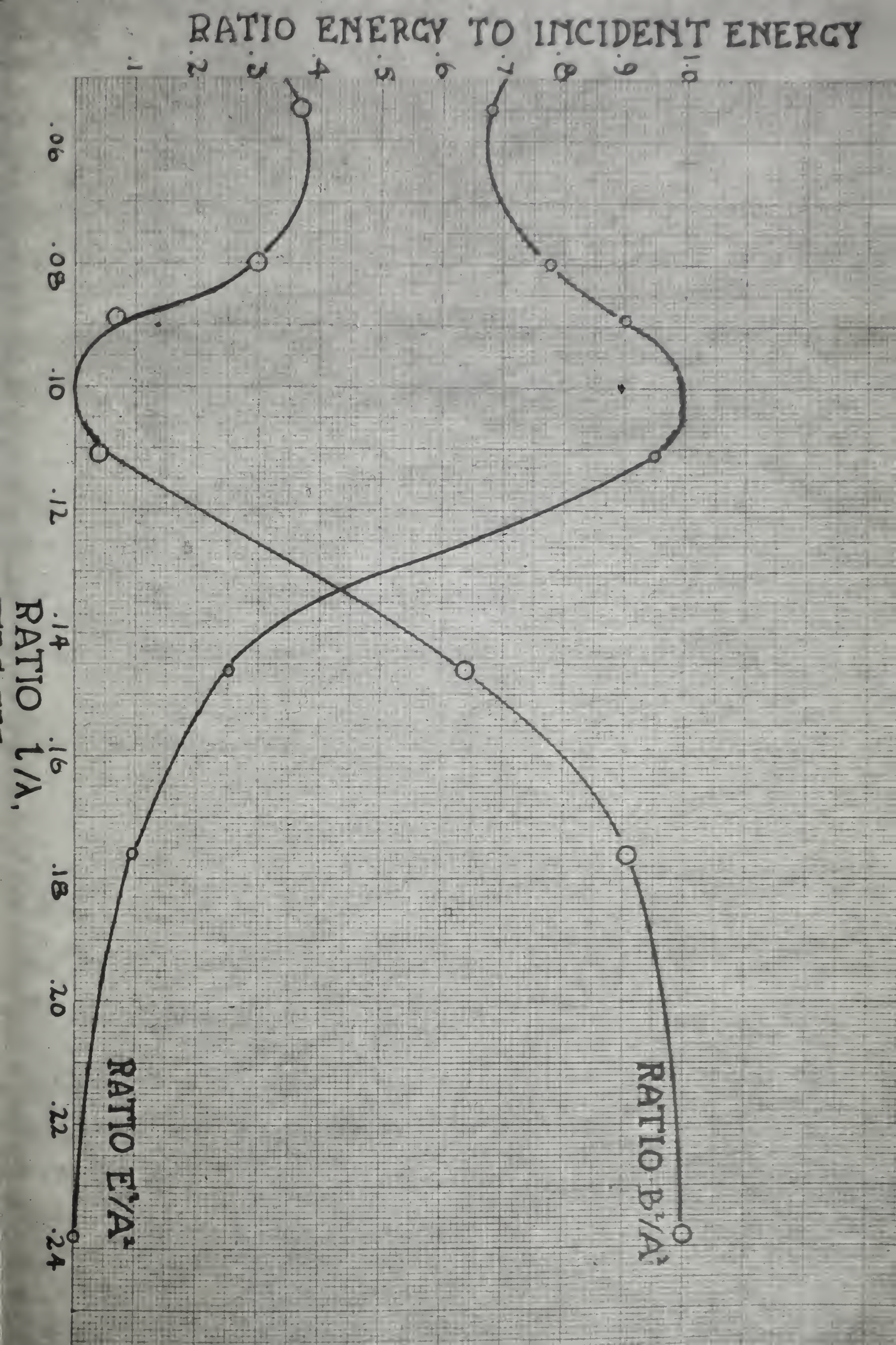
Moreover, on placing the next three thicker plates in the beam at the same incident angle it was found that each transmitted <u>more</u> of the incident energy than the one before it, till the fourth plate allowed nearly 100 % of the energy through at that incident angle (50°).

The question was raised as to a possible ~~effect~~ influence on the results of diffuse reflection or scattering of the waves, but this was shown to be negligible, within the limit of accuracy of the experiment, by taking simultaneous readings of the transmitted and reflected portions of the beam. Allowing for the attenuation of the beam due to its conical shape, the sum of reflected and transmitted energy was shown to equal the incident energy. These results are shown in Table VI and Fig. VI. The readings for E^2 were obtained with a plate in position. Readings for A^2 represent the deflection without the plate intercepting a portion of the beam. B'^2 represents the actual deflection of the torsion pendulum at T.P.'. located in the side bay of the tank. In order to obtain the value of reflected energy B^2, which would correspond to the energy of the reflected wave at the same distance from the transmitter as the pendulum which

TABLE VI

Showing ratio of transmitted to incident and reflected to incident energy for glass plates at 45° angle of incidence.

	E^2	A^2	B'^2	B^2	E^2/A^2	B^2/A^2	ℓ/λ
1	28	41	11	15	.68	.37	.055
2	32	41	9	12.3	.78	.30	.080
3	38	42	3	4.2	.90	.071	.089
4	42	44	1	1.4	.95	.032	.111
5	11	44	20	28.0	.25	.640	.146
6	4	42	27	38.0	.095	.91	.176
7	0	49	35	49.0	.000	1.000	.238


RATIO ENERGY TO INCIDENT ENERGY
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
RATIO L/λ
RATIO B²/A²
RATIO E²/A²

measured the transmitted energy, it was assumed that reflection was total for plate 7 and so the ratio $B^2:B'^2$ would be 49:35. Hence the column for B^2 was obtained from B'^2 by multiplying by this factor. The ratio ℓ/λ_1 was obtained by assuming a velocity of 5.8×10^5 cms./sec. and calculating from $\lambda_1 = \frac{V_1}{f}$. This experiment was carried out for the 45^0 angle of incidence only.

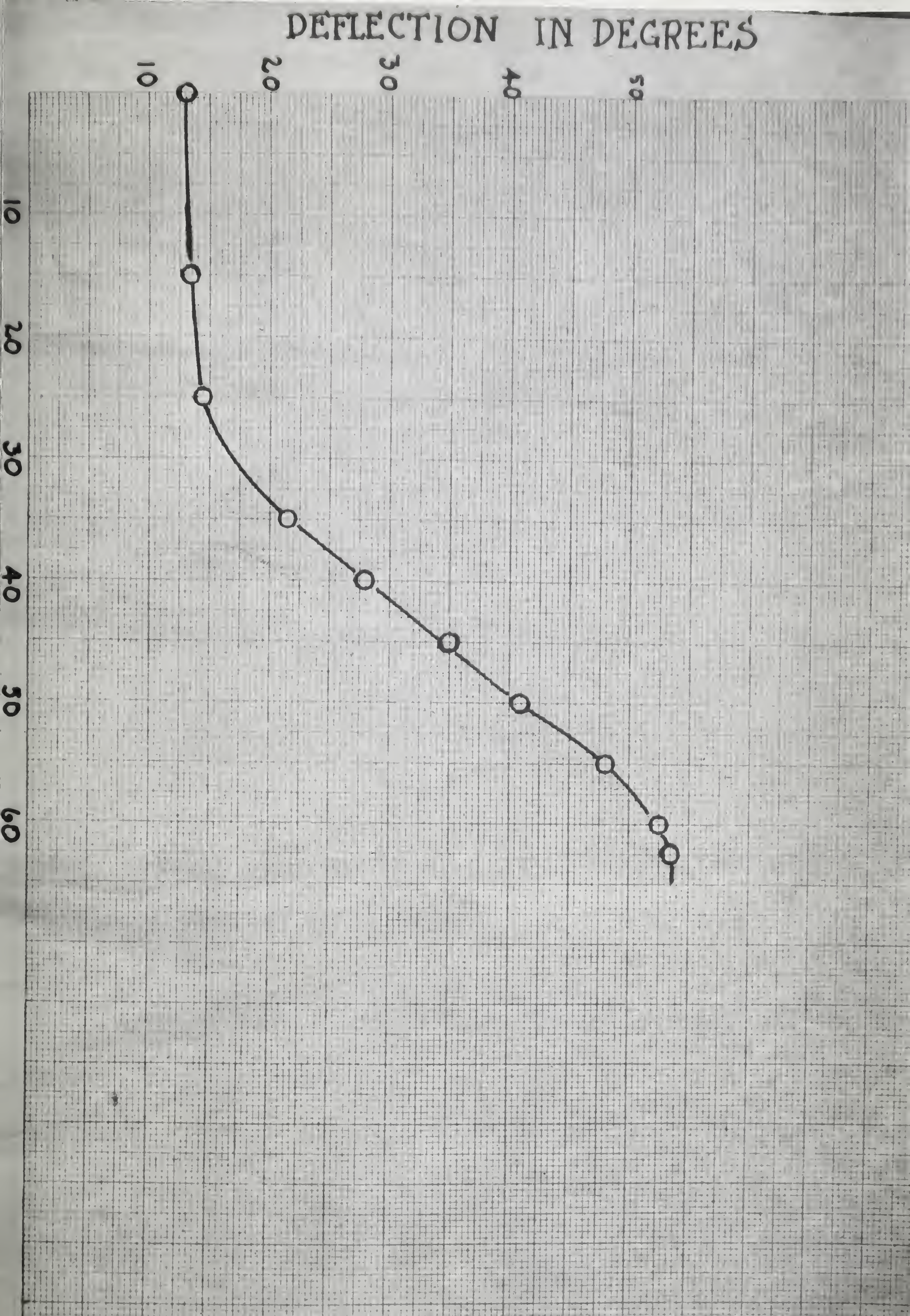
In order that some notion might be formed of the transmission phenomena occurring at other angles of incidence, a series of readings was taken for each plate, keeping the frequency and intensity constant and varying the angle of incidence from 0° to 62.5°.

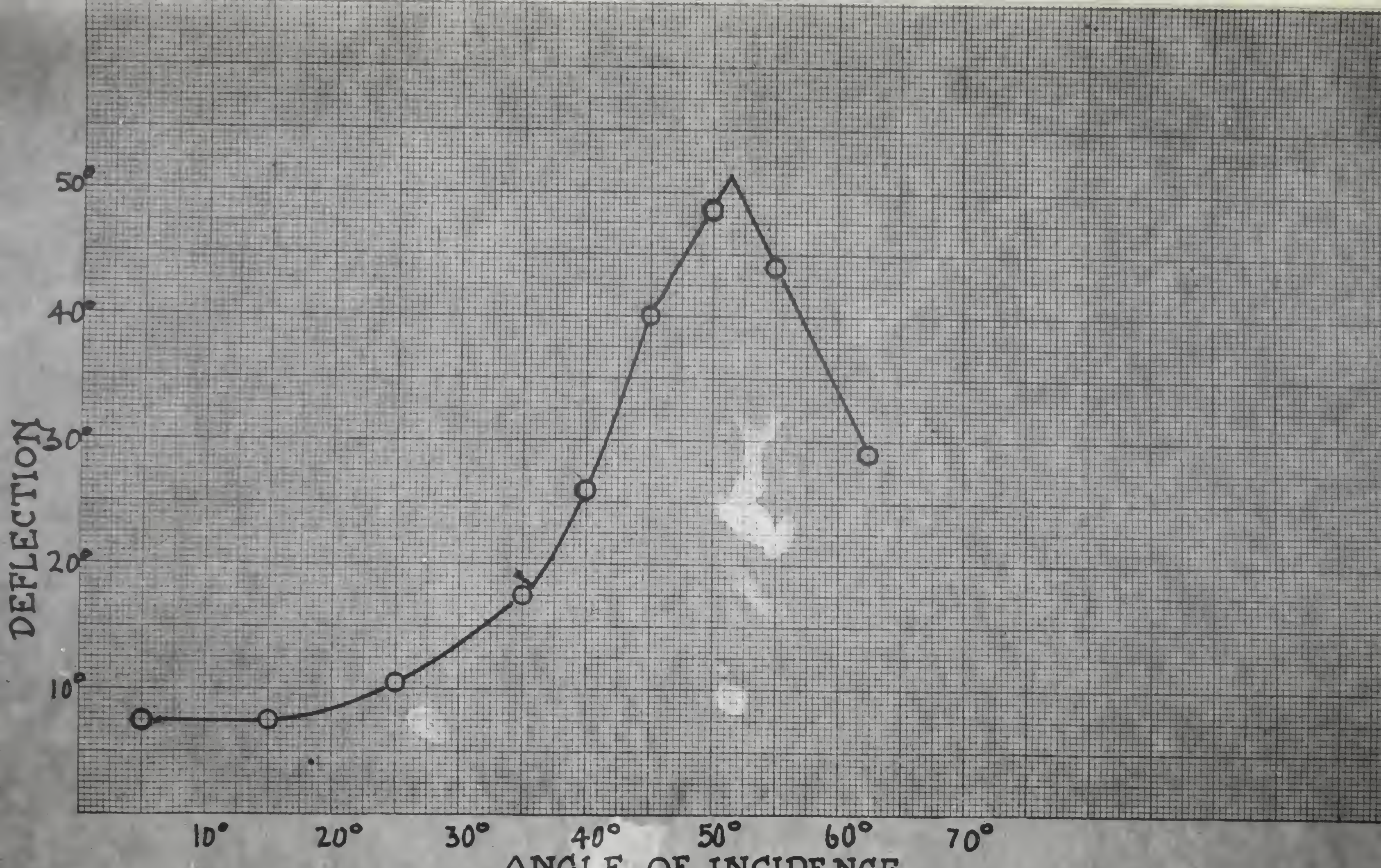
As may be seen by reference to Figures VII to XVIII and Tables VII to XVIII each plate shows maximum transmission for a certain angle, <u>this angle being greater and the peak broader for the thinner plates</u>.

This series was repeated at a frequency of 299,000 ~/sec. The results were much the same as before, except for the last two plates. As is shown in Fig. XVIII there are two angles for maximum transmission for Plate 7, and a distinct "plateau" on the slope of the curve for the second plate.

Figure XVII for Plate 6 shows only one distinct peak, but several irregular maxima between 21° and 0°.

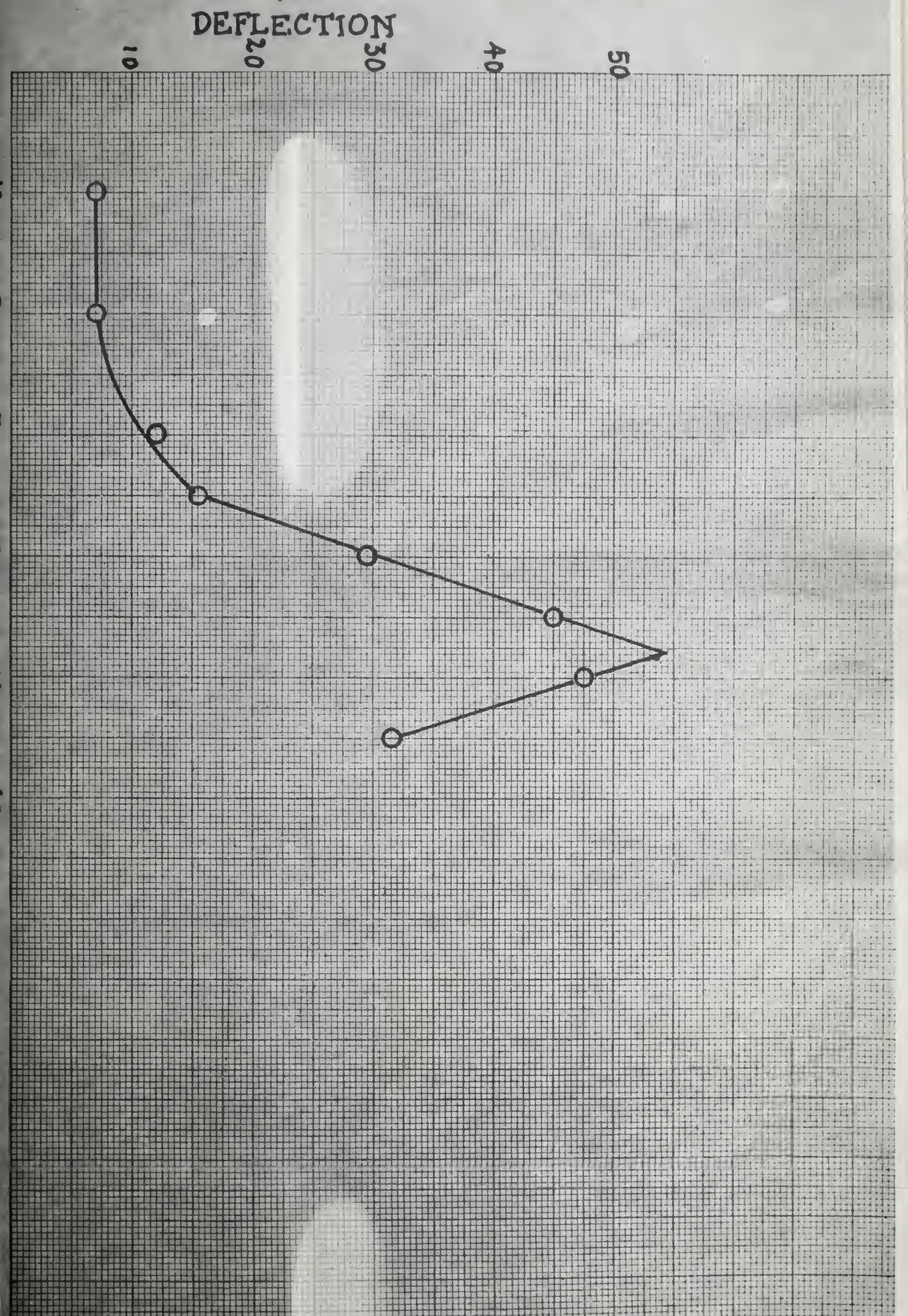
The foregoing tables and curves contain all the empirical data obtained; but this information, concerning the behaviour of a wave passing through a plate thin in comparison with a wavelength, may be rearranged graphically to yield further information concerning the factors determining the transmission of a wave under such circumstances. The experiments on transmission of wave energy normally through plates proved

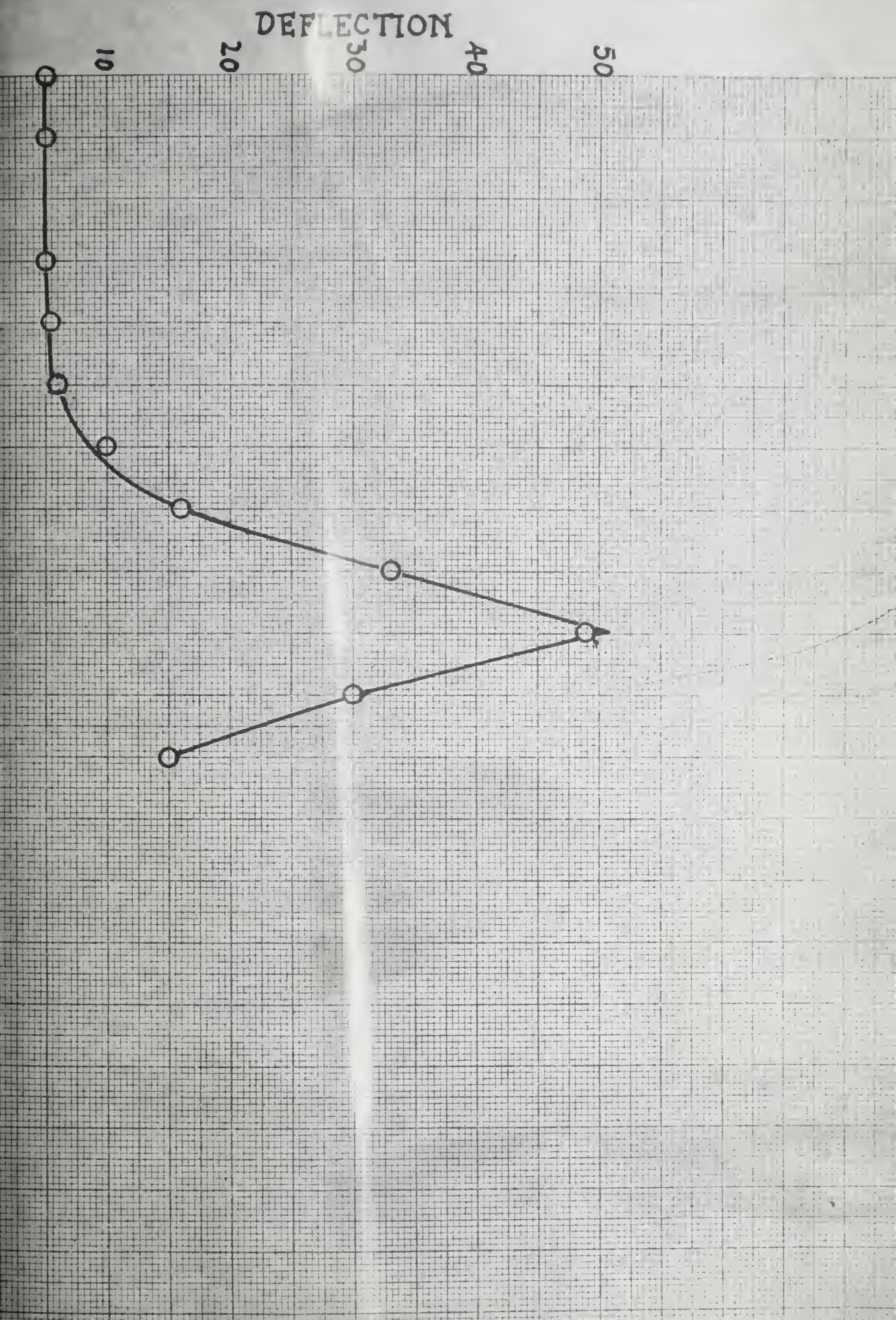

DEFLECTION IN DEGREES
10
20
30
40
50
10
20
30
40
50
60



DEFLECTION
10°
20°
30°
40°
50°
10°
20°
30°
40°
50°
60°
70°
ANGLE OF INCIDENCE

FIG. VIII


DEFLECTION
10
20
30
40
50


DEFLECTION
10
20
30
40
50

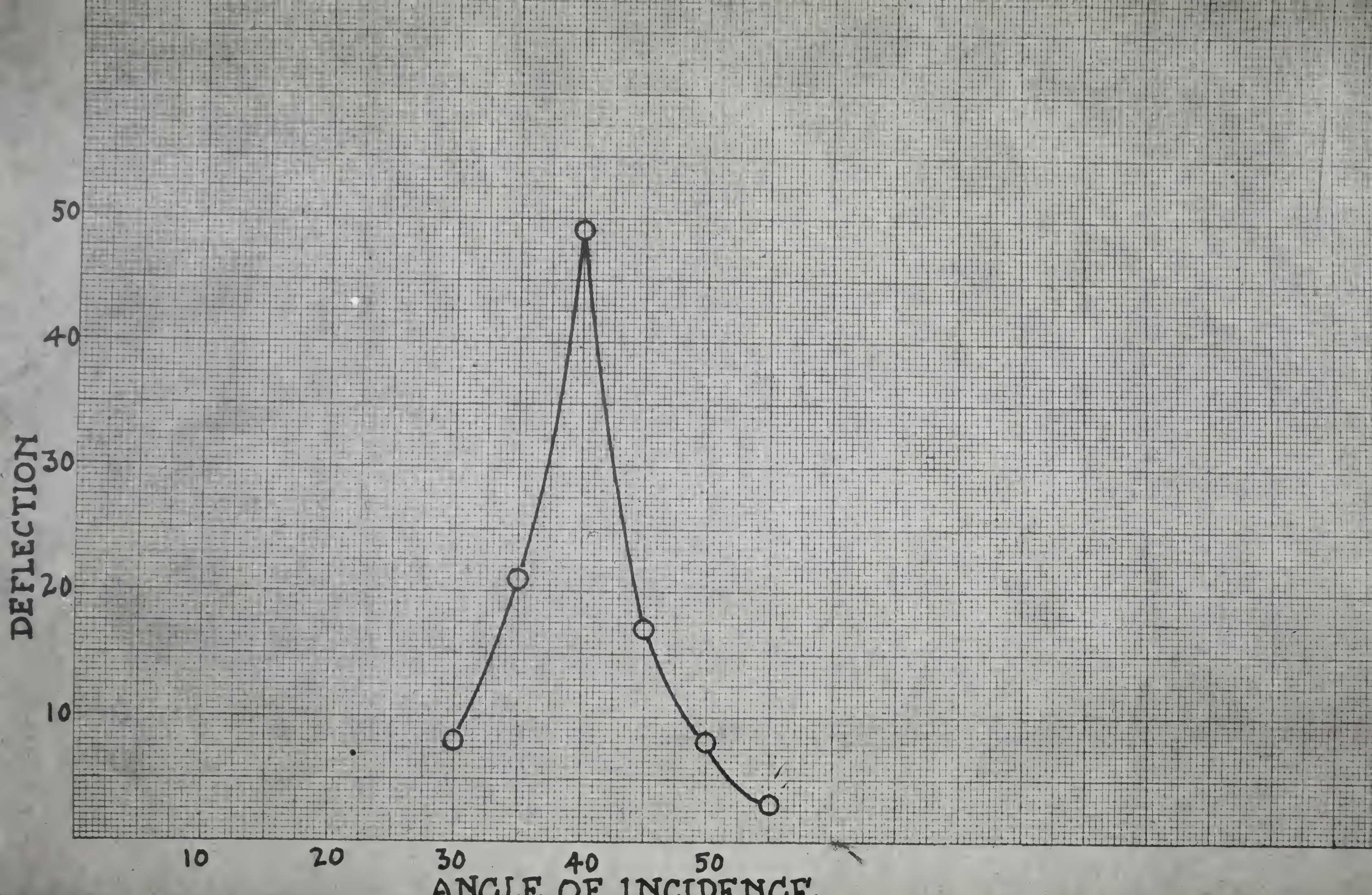
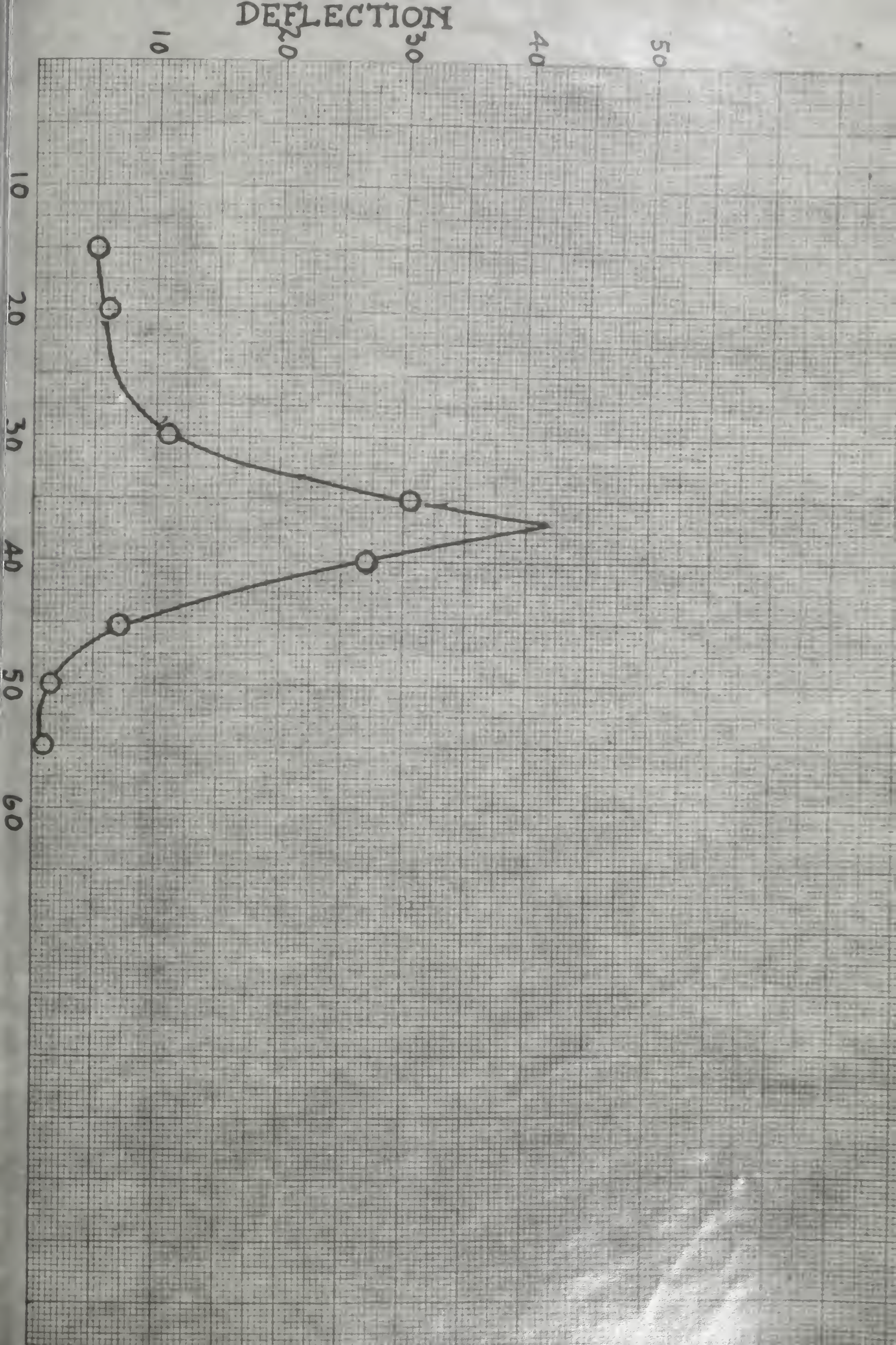
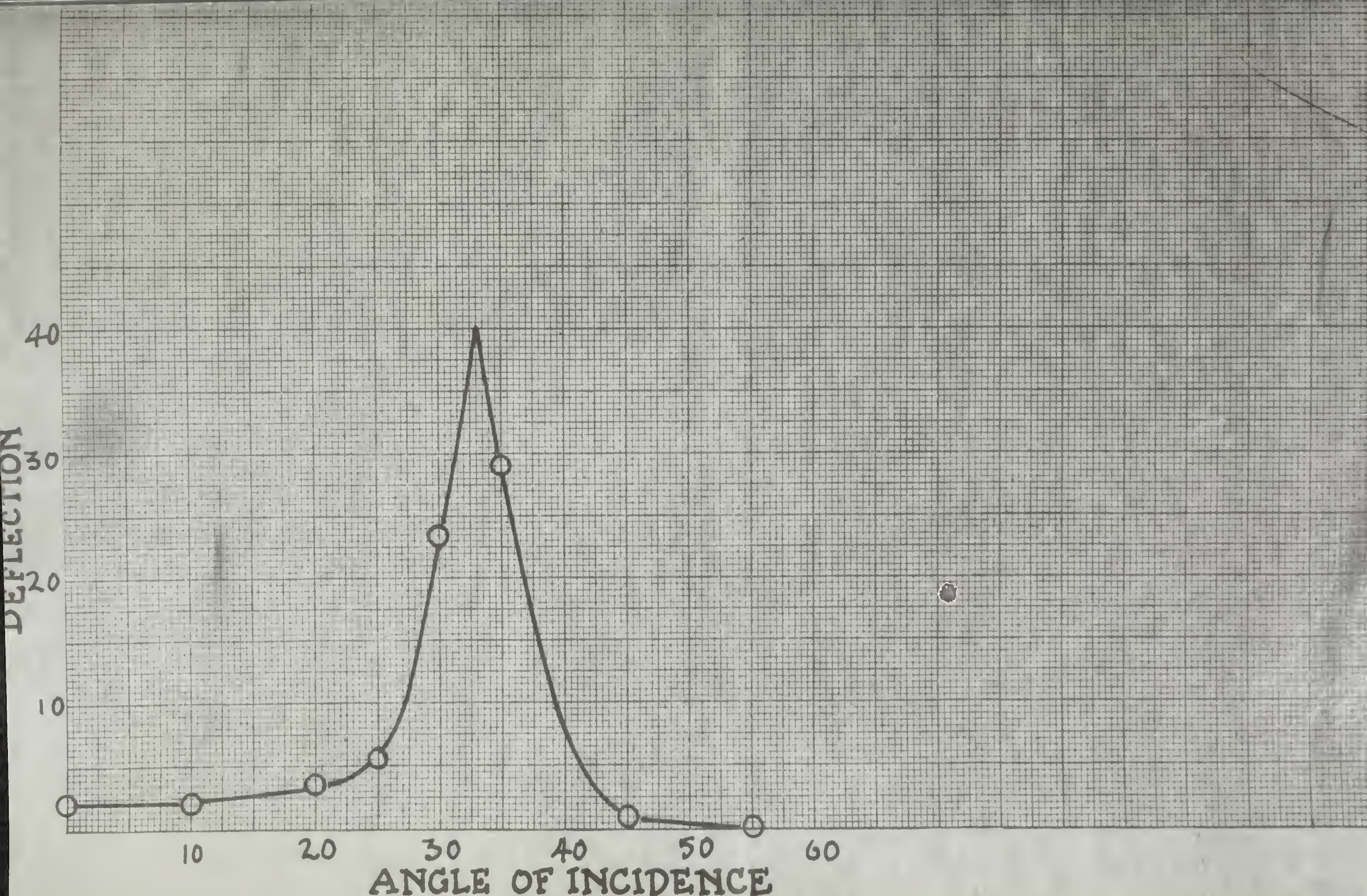
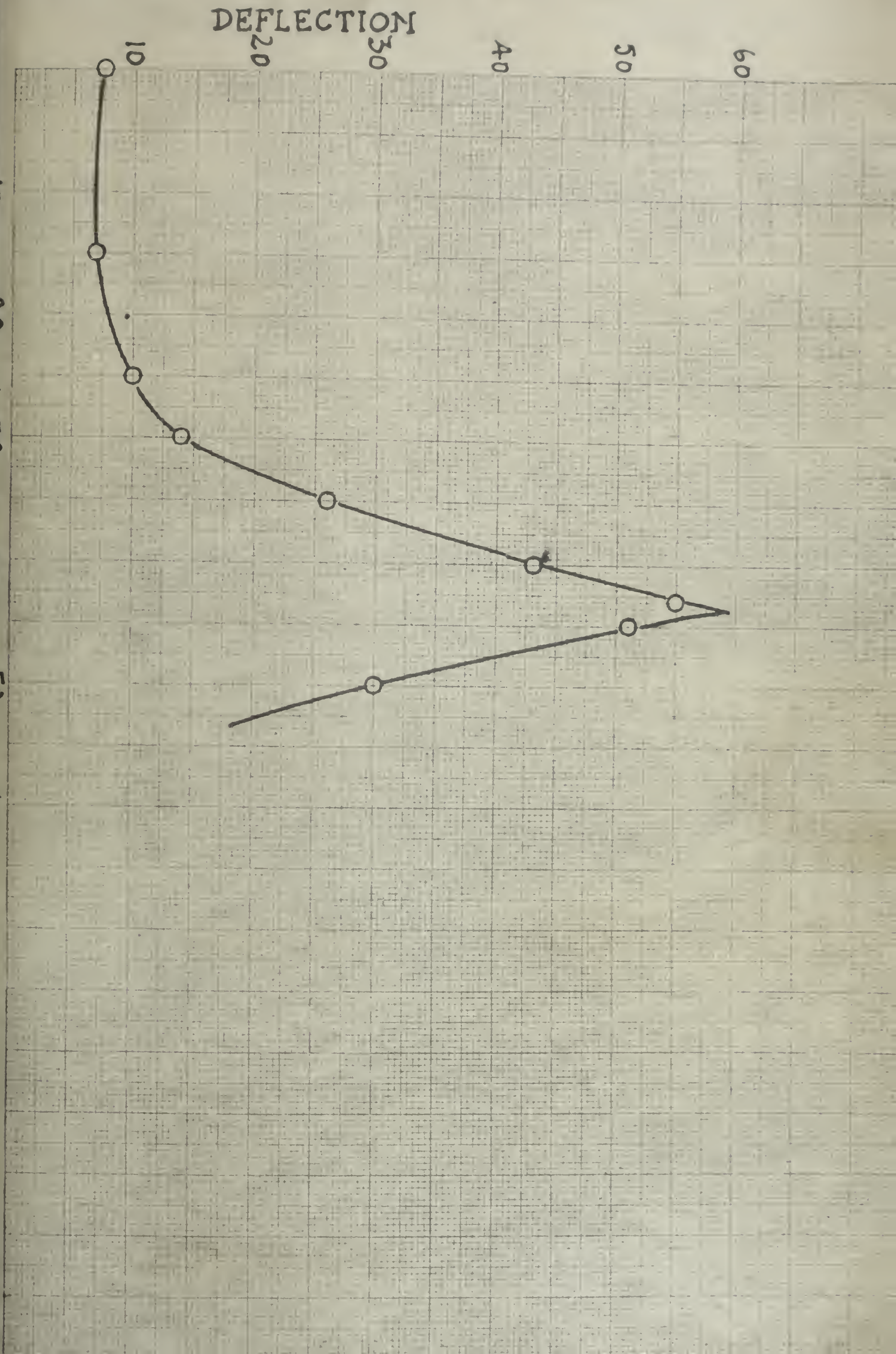


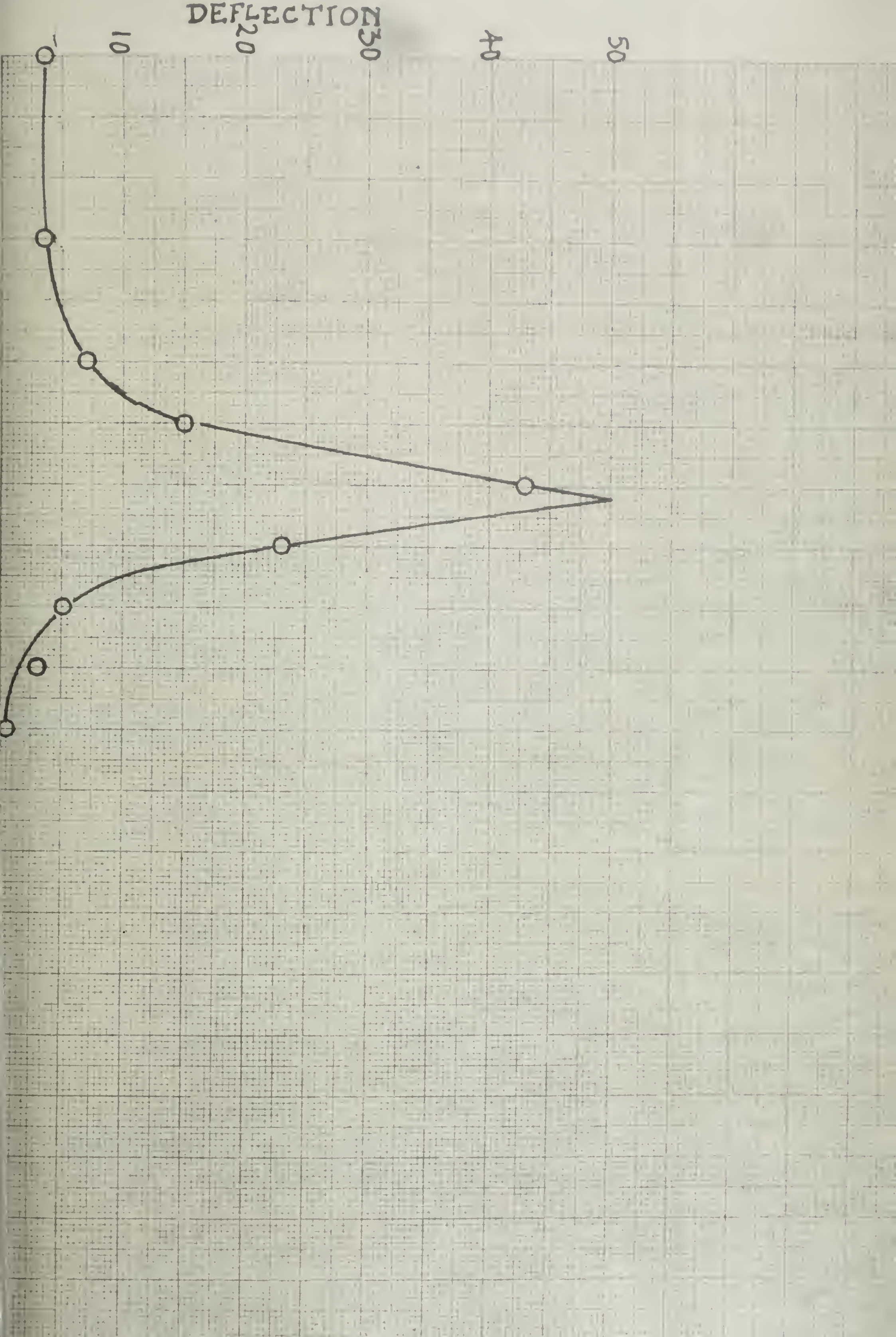
FIG. XI

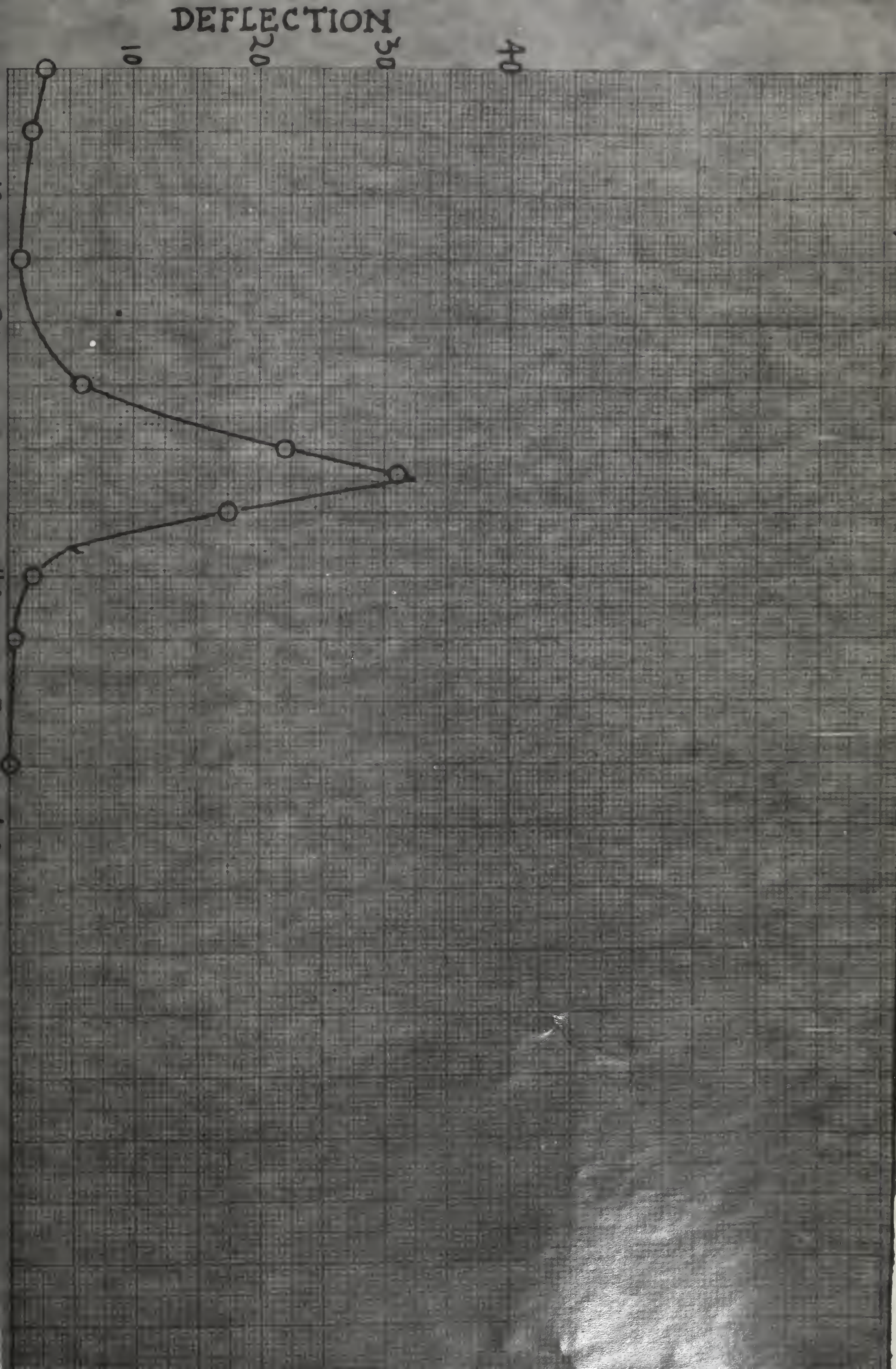


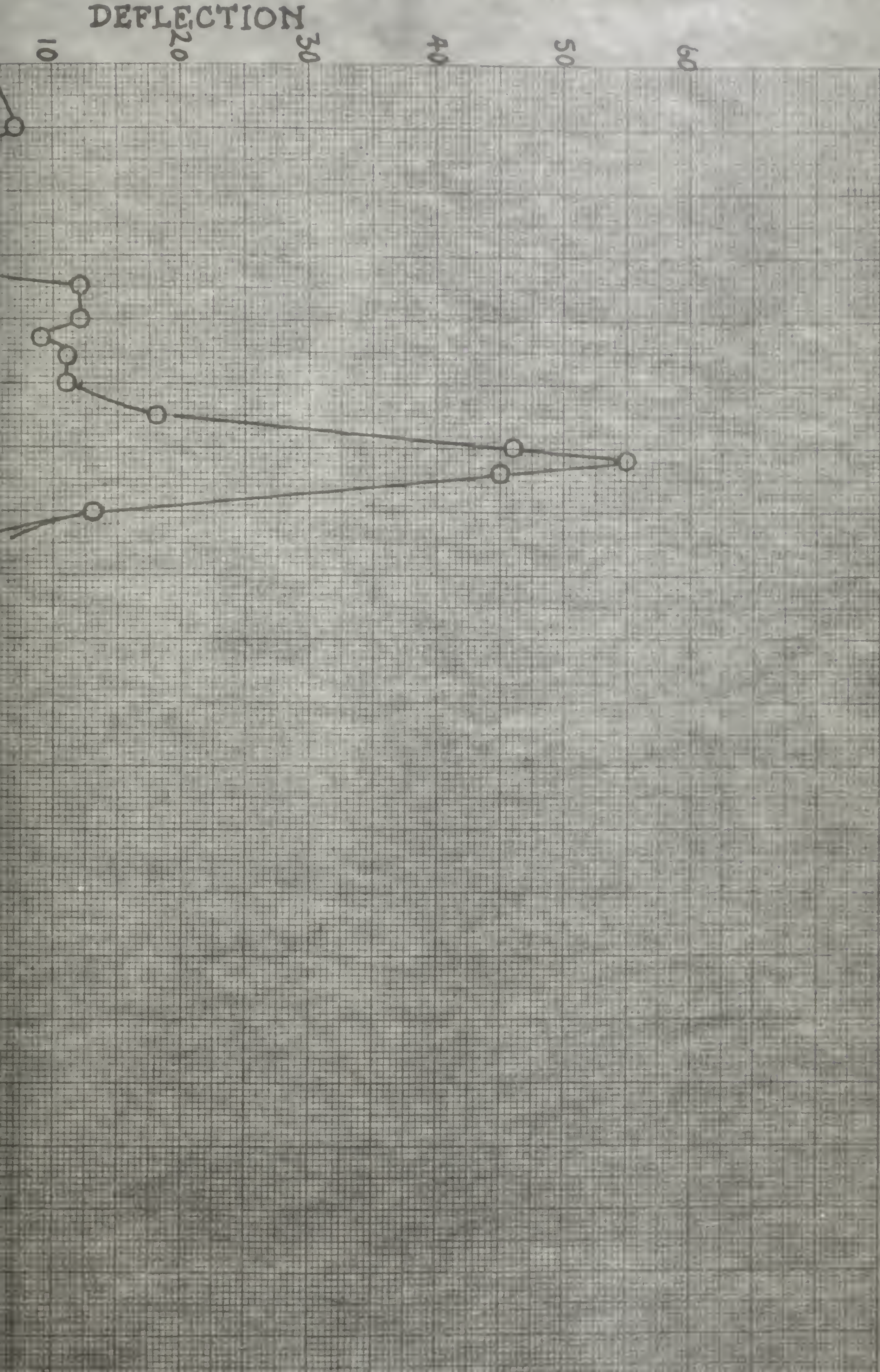
DEFLECTION
10
20
30
40
50
10
20
30
40
50
60

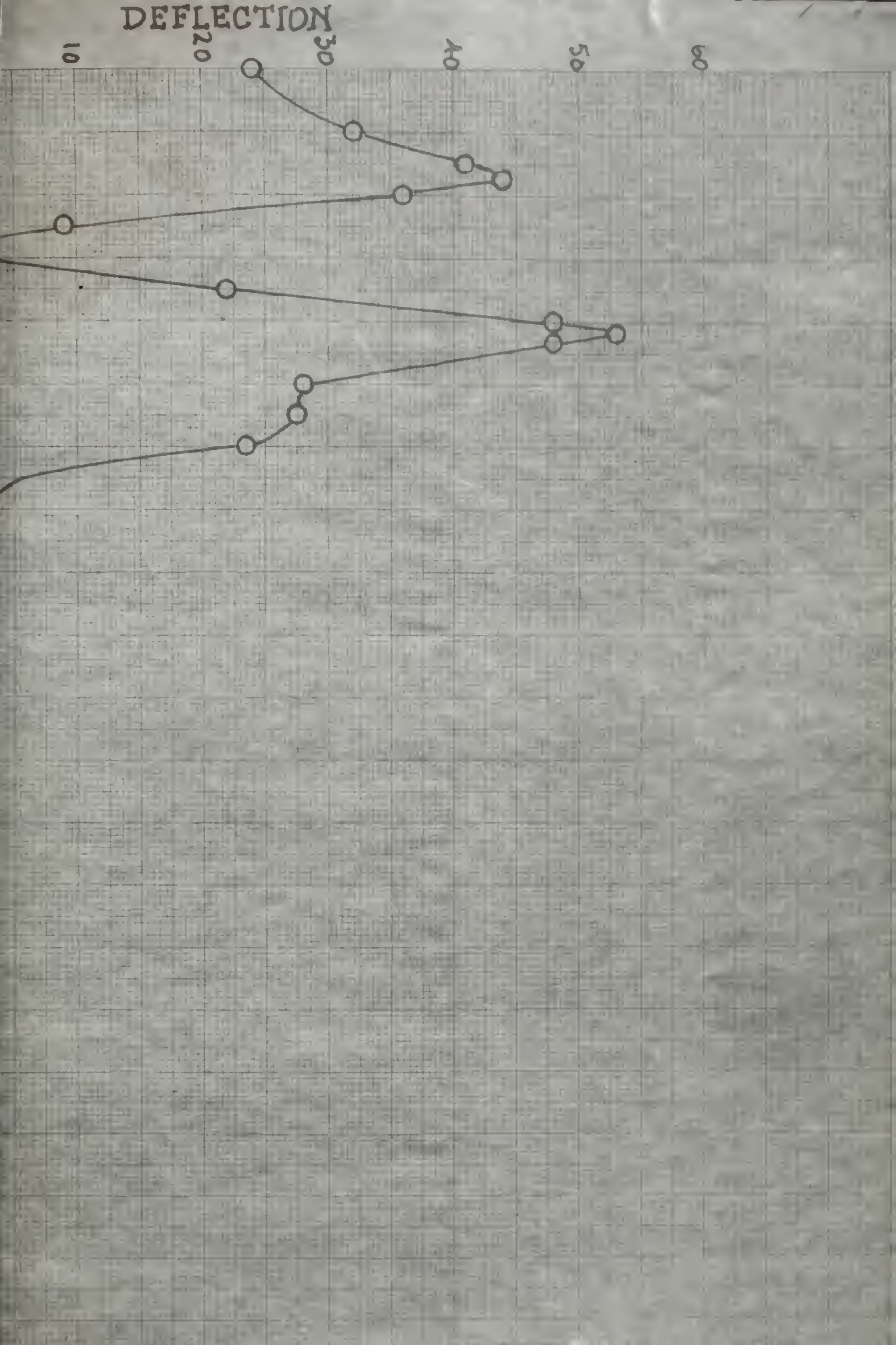

40
30
20
10
DEFLECTION
10
20
30
40
50
60
ANGLE OF INCIDENCE


DEFLECTION
10
20
30
40
50
60
50


DEFLECTION
0
10
20
30
40
50


DEFLECTION
10
20
30
40


DEFLECTION
10
20
30
40
50
60


DEFLECTION
10
20
30
40
50
60

Tables VII to XIII show transmission for Plates 1 to 7 at various angles of incidence and at a frequency of 157000~/sec.

TABLE VII - Plate 1

Angle of Incidence	Deflection
0	13.0
15	13.5
25	14.5
35	21.5
40	28.0
45	35.0
50	41.0
55	48.0
60	52.5
62.5	53.5

TABLE VIII - Plate 2

Angle of Incidence	Deflection
5	7.5
15	7.5
25	10.5
35	17.5
40	26.0
45	40.0
50	48.5
55	44.0
60	33.0
62.5	29.5

TABLE IX - Plate 3

Angle of Incidence	Deflection
10	7.0
20	7.0
30	12.5
35	15.5
40	29.5
45	45.0
50	47.5
55	31.5

TABLE X - Plate 4

Angle of Incidence	Deflection
00	5
5	5
15	5
20	6
25	6
30	10
35	16
40	33
45	49
50	30
55	15

[illegible]

TABLE XI - Plate 5

Angle of Incidence	Deflection
30	8
35	21
40	49
45	17
50	7
55	3

TABLE XII - Plate 6

Angle of Incidence	Deflection
15	5.1
25	5.5
30	11.0
35	30.5
40	27.0
45	7.0
50	1.4
55	1.0

TABLE XIII - Plate 7

Angle of Incidence	Deflection
0	2
10	2
20	4
25	5.5
30	18.5
35	29.5
45	1.0
55	0.0

Tables XIV to XVIII inclusive show transmission for Plates 1, 3, 5, 6, 7 at various angles of incidence and at a frequency of 299000 ~/sec.

TABLE XIV - Plate 1

Angle of Incidence	Deflection
0.0	7.5
0.5	7.0
25.0	10.0
30.0	14.0
35.0	20.0
40.0	43.0
43.0	55.0
45.0	51.0
50.0	30.0
55.0	20.5

TABLE XV - Plate 3

Angle of Incidence	Deflection
0.0	3.5
0.5	3.5
25.0	6.0
30.0	15.0
35.0	43.0
40.0	23.0
45.0	5.0
50.0	3.0
55.0	0.5

TABLE XVI - Plate 5.

Angle of Incidence	Deflection
0	3.0
5	2.0
15	1.0
25	6.0
30	22.0
32	31.0
35	17.0
40	2.0
45	0.5
55	0.0

TABLE XVII - Plate 6

Angle of Incidence	Deflection
0.0	4.5
5.0	7.0
7.5	2.0
10.0	1.5
12.5	1.5
15.0	1.0
17.5	12.0
20.0	12.0
21.5	9.0
23.0	11.0
25.0	11.0
27.5	18.0
30.0	46.0
31.0	55.0
32.0	45.0
35.0	13.0
40.0	2.0

TABLE XVIII - Plate 7

Angle of Incidence	Deflection
0.0	24.0
5.0	32.0
7.5	41.0
9.0	44.0
10.0	30.0
12.5	9.0
15.0	2.0
17.5	22.0
20.0	28.0
20.75	53.0
21.5	48.0
25.0	28.0
27.5	27.5
30.0	23.5
35.0	2.0
40.0	0.0
45.0	0.0
50.0	0.0

that for a given material the ratio was a function of ℓ/λ_1. That such is true in this case also is shown in Fig. XIX, which gives the ratio of transmitted to incident energy plotted against ℓ/λ_1 for an angle of incidence of 37°. The ratio is multiplied by the constant factor 90 for purposes of plotting.

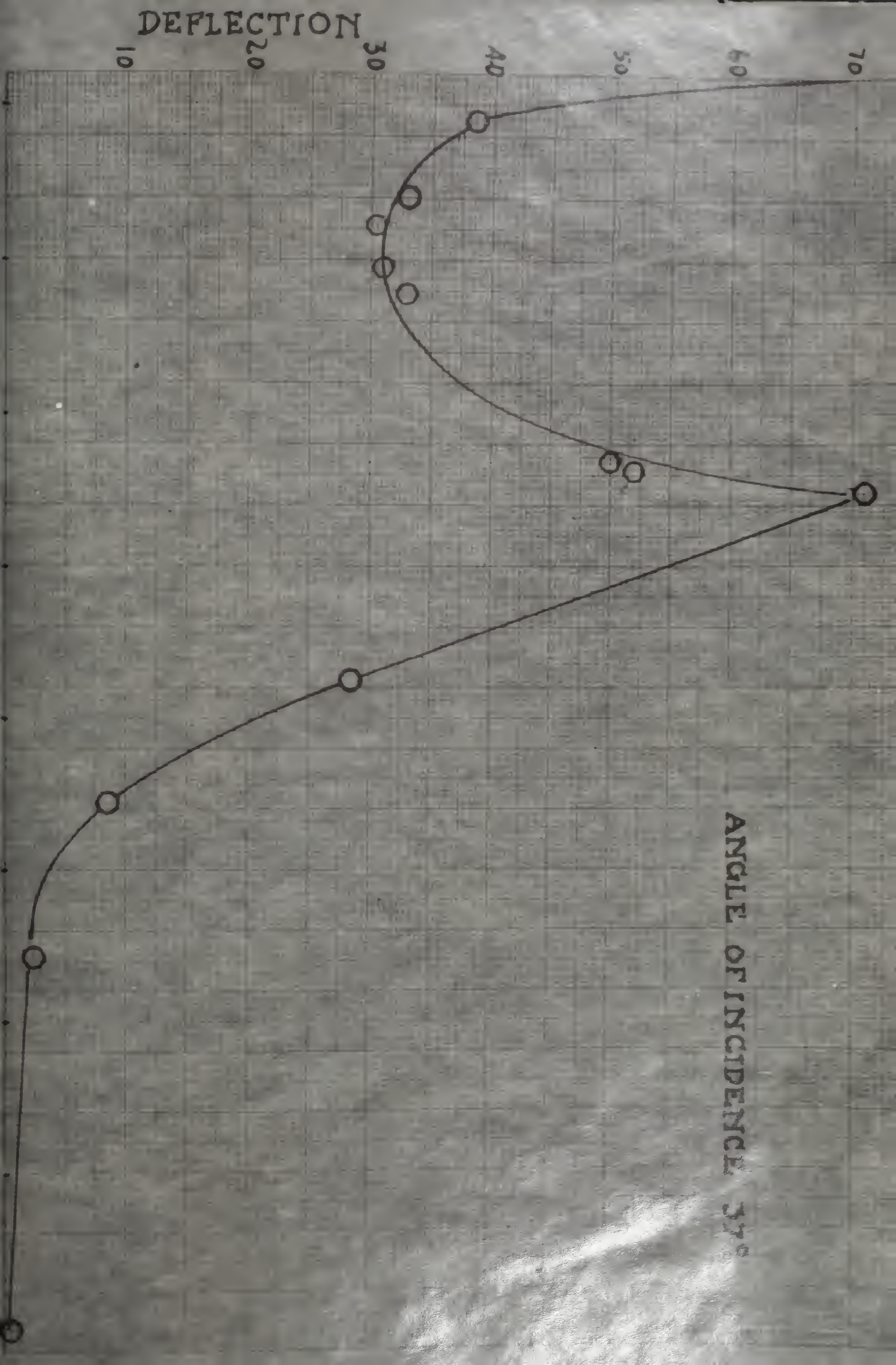
Fig. XIX offers conclusive proof that the expression developed by Rayleigh and quoted on Page 17 ~~is erroneous.~~ cannot satisfy the case. This theory cannot possibly account for the sudden rise at $\ell/\lambda_1 = .176$, but demands a steady decrease in the ratio E^2/A^2 from the left to the right as is shown on Page 18.

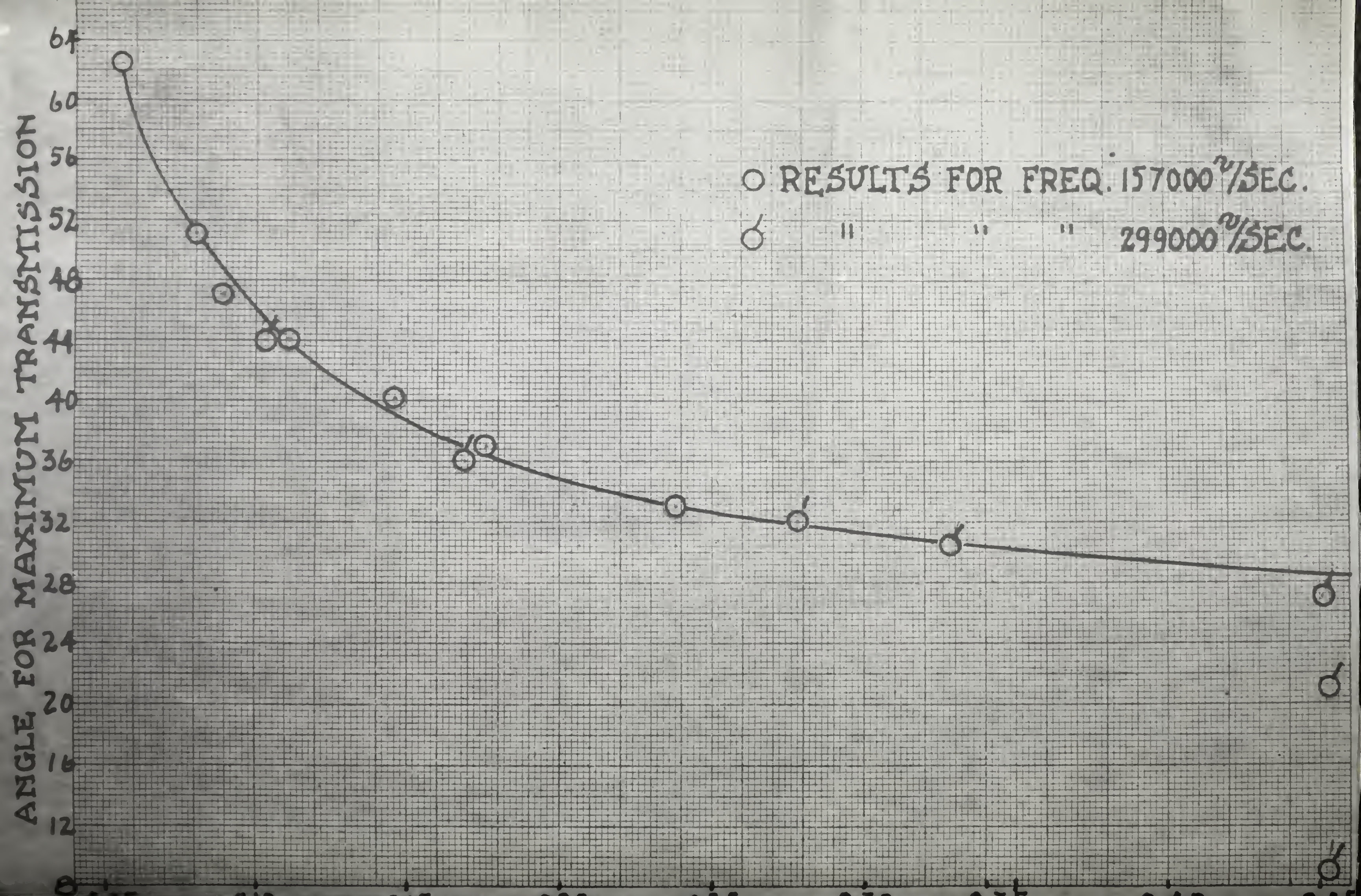
It was noted that the angle for maximum transmission for a given plate decreased as the thickness increased and also as the frequency increased. The significant factor in changing the frequency is the change in wavelength, so the decrease in the angle of incidence for maximum transmission might be expected to vary as the thickness and inversely as the wave length. That this is the case for those plates possessing only one peak of transmission is shown in Fig. XX.

Of the three points on the right hand side of Fig. XX the two lower ones were obtained by plotting the angle of each of the two peaks for Plate 7 against ℓ/λ_1, for that plate. The top point was obtained by treating the plateau on the side of the second peak (Fig. XVIII) as another peak. This latter point seems to fit more closely, but further work will have to be done before this can be stated with any degree of certainty.

Plate 7 (Fig. XVIII) shows a peak at 9°, angle of incidence. As this is less than the critical angle, Rayleigh's expression should hold. If it does hold, then we may apply the formula

1 cos $_1$ = m /2 to find V, by making substitutions, as follows,


DEFLECTION
10
20
30
40
50
60
70
ANGLE OF INCIDENCE 37°



ANGLE FOR MAXIMUM TRANSMISSION
○ RESULTS FOR FREQ. 157000 ~/SEC.
" " " 299000 ~/SEC.
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
0.30
0.35
0.40

TABLE XIX

Showing transmission by plates of various thickness at a fixed angle of incidence of 37°.

Plate No.	l/λ_1	$E^2/A^2 \times 90$
1	.055	38.5
2	.080	33.0
3	.089	30.5
4	.111	33.0
5	.146	50.0
6	.176	71.0
7	.238	28.5
1	.103	31.5
3	.169	52.0
5	.278	8.6
6	.329	2.5
7	.453	0.05

TABLE XX

Plate No.	e/m	$\theta_{max.}$
1	.055	62.5
2	.080	51.0
3	.089	47.0
4	.111	44.5
5	.146	40.0
6	.176	37.0
7	.238	33.0
1	.103	44.0
3	.169	36.0
5	.278	32.0
6	.329	30.5
7	.453	27.0
		21.0
		9.0

$$\lambda_1 = \frac{V_1}{n}$$

also $\sin\theta_1 = \frac{V_1}{V}\sin\theta$

and $\cos\theta_1 = \sqrt{1-\sin^2\theta_1} = \sqrt{1-\frac{V_1^2}{V^2}\sin^2\theta}$

~~also~~ Substituting in $l\cos\theta_1 = \frac{m\lambda_1}{2}$

$$l\sqrt{1-\frac{V_1^2}{V^2}\sin^2\theta} = \frac{mV_1}{2n}$$

~~and~~ Collecting and solving for V_1

$$V_1 = \frac{l}{\sqrt{\frac{l^2\sin^2\theta}{V^2}+\frac{m^2}{4n^2}}}$$

Taking l = .88 cms., θ = 9°, v = 1.48×10^5

the above expression gives V_1 = 4.5×10^5 cms./sec.

The quoted values for the velocity of sound in glass are all higher than this. Quoted values are generally based on results obtained from velocity determinations in thin rods, for which case the velocity is given by the familiar formula $V^2 = E/\rho$ where E = Young's modulus and ρ = density. Moreover, the velocity determinations based on the transmission of sound by plates at normal incidence show that such a plate may be treated as an infinite medium in which the velocity is given by Barton [11] as

$$V^2 = \frac{(1-\sigma)}{(1+\sigma)(1-2\sigma)}\cdot\frac{E}{\rho} \qquad \text{XII}$$

where σ is Poisson's Ratio

By a method to be described later, (Part II) $\sqrt{E/\rho}$ = 5.25×10^5 cms/sec. for ordinary window glass. Taking the lowest quoted value of Poisson's Ratio in glass as 0.229 (Lamb's Dynamical Theory of Sound, p. 113) and substituting in (XI) we get V = 5.65×10^5 cms./sec.

Thus it may be seen that whether the plate be treated as an infinite medium or as a rod, the velocity should be greater than 4.5×10^5 cms./sec., and this may be taken at least as an indication that the theory upon which this determination of velocity is based is in error. Hence it would seem that existing theory does not hold for very thin partitions for any oblique angle of in-

Taking λ = .05 [illegible] = [illegible], = .75, [illegible] = [illegible]

the [illegible] expression gives [illegible] = [illegible] cm.

[illegible]

[illegible]

(23)

[illegible]

[illegible]

cidence, whether greater than the critical angle or not.

It is also true that <u>the theory cannot account for a peak of transmission for a plate less than $\lambda/2$ in thickness at any angle of incidence</u>.

For, consider the expression which gives θ_1 for maximum transmission.

$$l \cos\theta_1 = \lambda_1/2$$
$$\cos\theta_1 = \lambda_1/2l$$
$$\text{If } l < \lambda_1/2 \text{ then } 2l < \lambda_1 \text{ and } \cos\theta_1 > 1$$

Various simplifying assumptions may be made in an attempt to find a satisfactory explanation.

1. Assume there is no refraction the transmitting layer of glass being so thin, but that different velocities exist in the two media.

In this case the retardation of phase in the passage of a wave is given by $2l\cos\theta - 2V_1 l/V \tan\theta \sin\theta$

The condition for total transmission in such a case is that the total retardation must be equal to an integral number of half wave lengths or

$$2l/\cos\theta - 2V_1 l/V \tan\theta \sin\theta = m\lambda_1/2 = mV_1/2f : \quad \text{Solving for } V_1$$

$$V_1 = \frac{2l}{\cos\theta\,(m/2l + 2l/V \tan\theta \sin\theta)}$$

By substituting $\tan\theta = \sin\theta/\cos\theta$, taking the denominator out of brackets, and dividing numerator and denominator by 2, we obtain

$$V_1 = \frac{l}{m\cos\theta/4l + l\sin^2\theta/V} \quad \text{........ XIII}$$

Taking m = 1, and substituting values of l, f and v for the first three plates gave velocities ranging from 1.13×10^5 cms./sec. to 1.34×10^5 cms./sec.

These values are too far from the expected values to afford much hope that the assumption is justified.

2. Assume that the velocity is not a constant, but varies and is determined by the equation given on p. 2~~4~~1(XI)

$$V_1 = \frac{\ell}{\sqrt{\frac{\ell^2 \sin^2\theta}{V^2} + \frac{m^2}{2\ell^2}}}$$

Making m = 1 and substituting values of ρ, v and f for the first five plates gave velocities ranging from 0.555×10^5 cms./sec. to 1.09×10^5 cms./sec. Again the values are too low to lend support to this assumption.

This problem for the time being will have to be left here, but it will be useful, when the work is resumed, to include a review of the experimental and theoretical work on the transmission of sound by plane partitions. This reveals the following:

1. Existing theory represents fairly well the propagation of a longitudinal wave through a plane partition at normal incidence. This has been shown to be true for plates as many as three half wave lengths thick, and for plates as thin as it is possible to machine on an ordinary turning lathe.

2. For ~~plates~~ partitions of glass and presumably other materials less than one half wave length thick, transmission of sound is not in accord with the theory for oblique angles of incidence greater than a certain angle, 9° in the case of glass. It is not known how small the angle of incidence must be or how thick the plate must be before the theory can be said to hold at all. Presumably the same kind of discrepancies would exist for any solid material.

3. Several simple modifications of existing theory have failed to account for the observed experimental facts.

4. Transmission by plates thin in comparison with a wave length is a function of the angle of incidence. The first derivative of this

function seems to be discontinuous at the peak of the curve. The function is as yet undetermined.

5. At a fixed angle of incidence, transmission is a function of l/λ. This function is also unknown.

Discussion.

It is at this point that the correctness of the statement made in the opening paragraph is most keenly felt. If it were possible to follow a wave in the medium through which it is passing it is almost inevitable that it would yield the clue to its ~~strange~~ behaviour. As matters are, it is only known that for a given angle of incidence a certain proportion of the incident energy does pass through a ~~plate~~ partition whether the energy is incident at angles less than the critical angle or not. Whether or not the ~~plate~~ partition in some way transforms the incident longitudinal disturbance into a transverse or torsional vibration and then passes it on again in its original form is not known, and cannot be determined by present experimental technique.

This inability to follow the progress of a wave step by step is also characteristic of present analytical methods. We may assume certain boundary conditions, and while these assumptions may yield results which are verified by experiment, yet the precise actual nature of the stress and strain phenomenon in the medium remains unknown.

This quandary may be entirely due to the classical point of view which treats a solid material as a continuous medium. Such a simplifying assumption is undoubtedly most useful in certain cases, e.g.

in the expression of a wave as a trigonometrical function, but must eventually lead to difficulty if always adhered to rigidly. It may be that at the present stage of development of ~~mathematics~~ analysis the only feasible assumption to make from the point of view of an investigator of a compressional wave is that solid matter is continuous. Even so, it is obvious that many advantages would be attendant upon the sssumption that gross matter is not continuous if it were possible to follow the energy of a compressional wave from molecule to molecule through a material by some analytical device. No obvious, simple method of representing even a simple sine wave through the solid in terms of molecular motion or motion of the crystal structural units suggests itself. However, if analytical methods can be developed which will accomplish the above, then it may easily be seen that the problem of transmission of a compressional wave through a plane partition at angles of incidence greater than the critical angle will offer no special difficulties. In fact if the fundamental factors involved in the transmission of energy from one molecule to another can once be solved, it is almost inconceivable that any problem in wave motion in solids could remain unsolved. The analytical procedure would be simply to start a wave in a medium and follow it through step by step.

The development of this point of view would probably be accelerated by developing a corresponding experimental method, for example, by using a large scale model of a crystal, in which waves could be started and followed through from unit to unit of the lattice structure from one instant to the next.

This is speculation, but other problems as well as the present one have indicated that some such change in the method of attack is needed. For this reason, the suggestion is included here.

Before any method of analysis can be developed to solve theoretically the problem on which experimental results are here collected, it will be necessary to state as explicitly as possible the factors involved.

1. Experiment and theory in optics and acoustics both indicate that thin films or partitions transmit the maximum amount of energy when the path difference between the directly transmitted and the refracted and reflected ray, is some multiple of $\lambda/2$. This path difference may be due to any one or all of the following factors:

(a) difference in distance travelled in the two media;

(b) change of phase at one or both surfaces;

(c) difference in velocity in the two media.

Considering these factors in connection with the present problem we discover not one but several reasons for the failure of theory to apply for angles of incidence greater than the critical angle.

As Rayleigh has said, when the angle of incidence exceeds the critical angle, the disturbance in the second medium is not a wave at all. It does not travel into the second medium, although a disturbance exists in this medium, and so (a) above seems to have no meaning. A wave which does not travel into the second medium can hardly be said to suffer a change of phase at the second surface, as it cannot advance to, nor be reflected by, that surface. The factor (c) also has a doubtful meaning on the assumption that the disturbance in the second medium has the properties assigned to it in the analysis, for there can only be a definite velocity in a material when a wave is permitted to form in it.

However, experiment shows that the primary wave does

travel through and out of the second medium. If we are to give any meaning to factor (a) we must have a recognizable wave which advances to and is reflected by the second surface. This indicates the necessity of a clearer view of what takes place in the phenomenon of refraction. It may be that the velocity of propagation of a wave is profoundly modified in a thin plate; that is, modified until the velocity in the plate is small enough to eliminate the phenomenon of total reflection. Present methods of analysis give no indication as to the why or wherefore of phase change under these circumstances, so factor (b) remains unknown.

A detailed consideration of the question of the velocity of propagation of a compressional wave in a solid yields several points of interest in connection with (c) above. For a given material, the velocity of a compressional wave depends on the potential energy of a given compression in the material. The potential energy of a given compression depends to some extent on the geometrical shape. For instance, the potential energy of a given compression in the interior of a limitless solid is greater than that due to a corresponding compression in a thin rod, as the lateral yielding in the latter case reduces the restoring force. A case intermediate between the two above is that of a thin plate. Here lateral yielding is possible in one direction only. Further, the potential energy due to a given compression is greater in the interior of a solid than at or near its surface. This accounts for the Rayleigh[10] wave, which travels along the surface of a solid with a velocity which is less than the velocity of the wave in the interior in the ratio of 0.92:1. This wave has been observed in the case of earthquakes, following in the wake of the wave travelling through the interior of the earth. In this case, the secondary wave is behind the first wave not only because it is the slower of the two but because it must travel in a circle while the faster wave

travels in a straight line.

Another factor which may modify the velocity of a wave in a material is that due to lateral inertia. Rayleigh calculated the effect of this for the case of a cylindrical rod thick in comparison with a wave length. A series of experiments[9] performed by the writers verified Rayleigh's expression.

Applying these facts to the present case; it may be seen that a wave travelling normally through a very thin plate will not be propagated with the same velocity as a wave travelling in a direction parallel to the surface of the plate, as the potential energy due to a given compression is not the same in these two cases. Further, a wave passing through the plate obliquely might be expected to be propagated with any velocity between the two extremes, depending on the angle of incidence. Moreover, a wave might easily be conceived as travelling with the velocity of the Rayleigh wave just after entering and just before leaving the plate, but with the velocity of a wave in an infinite medium while in the interior of the plate. Also, at certain angles of incidence at least, the velocity of the wave in the plate may be modified by an effect due to lateral inertia.

In conclusion of this part of the work, it may be said that an adequate theory for transmission of wave motion through very thin plates at angles of incidence greater than the critical angle must provide for and abide by the following:

1. Almost total transmission for each plate at some particular angle of incidence, this angle being greater for thinner plates.

2. Almost total transmission at each angle of incidence for plates of some particular thickness, this thickness being greater for smaller angles of incidence.

[9] Boyle and Sproule, Rep. Can. Res. Council - May 1928. ~~April 1929~~.

3. Theory must not implicitly forbid the existence of a wave in the second medium for angles of incidence greater than the critical angle.

4. Theory may have to allow for, and determine the magnitude of a possible phase change other than zero or π at one or both surfaces.

5. Theory will almost certainly have to allow for and determine the magnitude of different velocities of propagation in different directions in the plate. These differences may be introduced by virtue of differences in elastic constants in different directions, or by an effect due to lateral inertia. Further the velocity of propagation in a given direction may vary with the distance from the surface of the plate, giving, in effect, a continuously variable medium.

As has been suggested above, the solution of these problems may be beyond the scope of present methods of analysis and may necessitate the development of new analytical devices and the adoption of assumptions more in accord with experimental fact.

APPENDIX TO PART II

Visualization of Phenomena Occurring by Virtue of Transmission of Sound Waves Incident at Angles of Incidence Greater than the Critical Angle.

A method which has often proved its value in enabling investigators to obtain quickly a qualitative idea of phenomena occurring in an ultrasonic beam, has again been found useful here.

A glass plate A - Fig. XX-C was suspended in the beam at an angle of incidence of 45°, as shown in Fig. V-A. Glass plate B, Fig. XX-C was suspended normally to the beam about 1.5 meters back of the centre of plate A, so that any of the beam transmitted through A would be reflected back on its path by B, forming a standing wave system.

The nature of this standing wave system was made evident by dropping sifted cinder~~s~~ dust into the water in the path of the beam. The cinders were caught on a sheet of thin cheese-cloth stretched on a metal framework shown in Fig. XX-C. The cinders tend to move into the nodes, so the arrangement of the cinders on the cloth may be taken as traces of the the arrangement of the nodes above the cloth.

Fig. XX-C shows the arrangement of the transmitting and reflecting plates and the cloth screen. Fig. XX-D shows a close-up of the dust figure obtained. The arrow indicates the direction taken by the beam.

Several points of interest may be discovered in the arrangement of the dust shown in these two figures.

1. The mere existence of arrangement is an indication that the beam is transmitted through the plate at angles of incidence greater than the critical angle.

FIG. XX-C

2. The velocity as calculated by a measurement of the distance between nodes is 1.48×10^5 cms./sec., which indicates that the transmitted wave is longitudinal. ~~It is difficult to conceive any other type of wave propagated by a liquid for any appreciable distance, yet such an occurrence is not impossible.~~

3. It may be observed that the dust is not only arranged in rows parallel to the reflecting plate B, but that over a considerable area near plate A the cinders are in rows parallel to plate A, and perpendicular to Plate A, parallel to plate B and perpendicular to plate B. This is simply due to the fact that plate A is acting as does a semi-silvered plate in optics. That part of the energy transmitted by A is reflected by B back to A where it is partly reflected. This reflected component produces a system of nodes parallel to A as in Wein's experiment in optics.

PART III.

Transverse Vibrations in Bars and Sheets

Owing to the doubt as to the actual velocity of a compressional wave in the glass used in the experiments in Part II an attempt was made to find it by an independent method.

A strip of common double diamond window glass 2 cms. wide, 91.6 cms. long and 0.303 cms. thick, was sealed to the end of an ultrasonic transmitter as shown in Fig. XX-A. This transmitter was not of the type ordinarily used to produce an ultrasonic beam, so it will be described in an appendix on transmitters. It will suffice here to say that vibrations could be set up in it with the same radio frequency oscillator as is used in other and usual experiments with the ultrasonic beam.

The flat surface of the glass was arranged horizontally, and sand placed on it. It was hoped that the longitudinal vibration in the transmitter would be propagated into the glass strip, and the standing waves produced in the strip would be made evident by the motion of the sand to the nodes of displacement. The wave length cou could then be found by measuring the distance between nodes, and the frequency determined by measuring the frequency of the electrical oscillations with a radio frequency wavemeter. The velocity could then be determined from the familiar formula $V = f \lambda$.

The rod was set in vibration at a frequency of 38000~/sec. Nodes were formed, indicating that some sort of wave was transmitted into the glass, but when the loops were measured and the velocity calculated it was found to be 1.05×10^5 cms./sec. while the quoted values for the longitudinal velocity in glass range from 5×10^5 cms./sec. to 6×10^5 cms./sec. A determination at a second frequency

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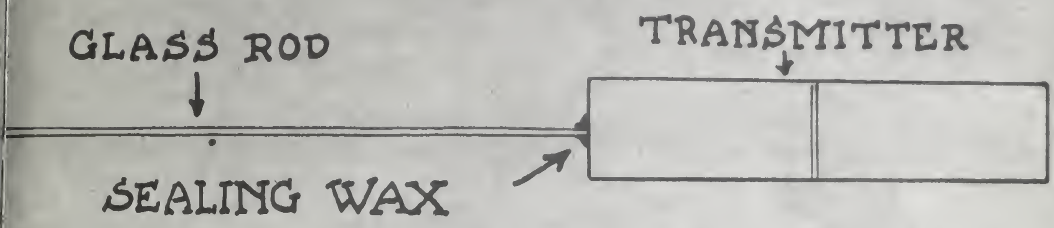

FIG. XX-A

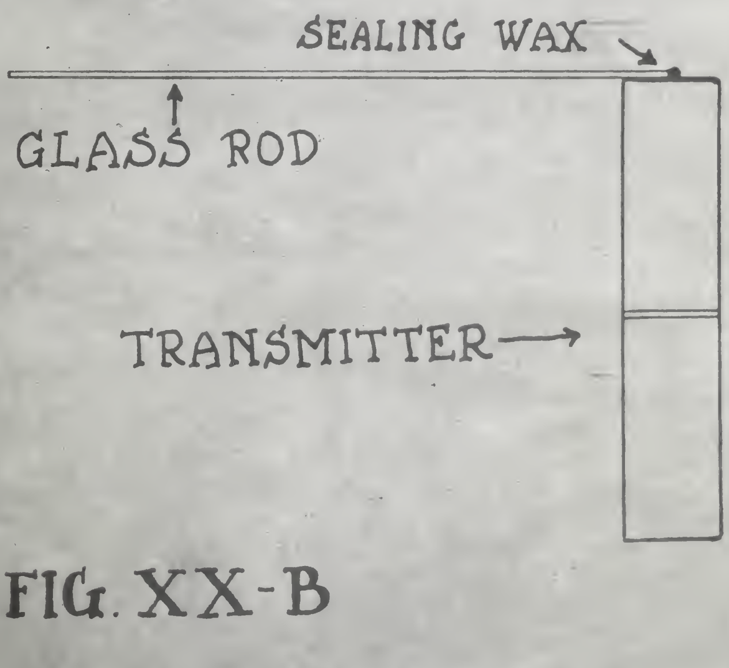

FIG. XX-B

of 20700 ~/sec. showed that the velocity was 0.775×10^5 cms./sec. This strengthened the opinion that the wave was transverse rather than longitudinal, for the velocity of a transverse wave is a function of the frequency, while that of a longitudinal wave is independent of frequency for a strip as narrow as the one here employed.

Lamb [~~11~~ 13] has given an expression for the velocity of propagation of a transverse wave in a bar

$$C^2 = \frac{K^2 k^2}{1 + K^2 k^2} \cdot \frac{\epsilon}{\rho} \qquad \text{XIV}$$

where C = velocity of transverse vibration

$K = \frac{2\pi}{\lambda}$ where λ = wave length

ϵ = Young's modulus

ρ = density

k = radius of gyration of the area of the cross-section about an axis through its centre of gravity, normal to the plane of flexure.

For a rectangular prism such as used here, $k^2 = a^2/12$ where a = thickness of prism or bar, measured in the plane of flexure.

Calculation based on (XIV) indicated that the vibration in the glass was flexural. The glass rod was then sealed to the transmitter as shown in Fig. XX-B. With such an arrangement the longitudinal vibration in the transmitter would necessarily set up flexural vibrations in the glass strip. Using the same frequencies as before, the nodes occurred at the same intervals as with the arrangement in Fig. XX-A. This gave final confirmation to the fact that energy supplied to a thin rod in the form of a longitudinal vibration is ~~totally~~ mostly transformed into flexural vibration. This has been foretold from theoretical considerations by Lord Rayleigh [~~12~~ 14]. The analogy between this case and Melde's experiment with a vibrating string is too

obvious to require comment.

Using the arrangement of apparatus as in Fig. XX-A, several attempts were made to damp out the flexural vibrations and at the same assist the longitudinal vibrations, as the experiment could not yield a direct determination of the velocity of a longitudinal wave unless that form of vibration could be made the predominant one. This effort failed, as the process of damping the undesired mode of vibration so reduced the longitudinal form that it could not be detected. The arrangement of apparatus shown in Fig. XX-B was finally made to yield the desired information in the following way.

The velocity of a longitudinal vibration in a thin rod is given by $V^2 = f/\rho$. Now f/ρ occurs in XIV, so that expression may be used to calculate V, for all the other factors in the expression may be measured.

As stated above, c may be determined by measuring the distance between the nodes. $C = f\lambda$. By definition $K = 2\pi/\lambda$, so K is also determined. The thickness of the strip is readily determined by means of a micrometer screw gauge.

XIV may be re-written as

$$C^2 = \frac{K^2k^2}{1+K^2k^2} V^2 \quad \text{or} \quad V^2 = \frac{C^2(1+K^2k^2)}{K^2k^2}$$

$$\text{or } V = \frac{c}{M} \quad \text{where } M = \left(\frac{K^2k^2}{1+K^2k^2}\right)^{\frac{1}{2}} \qquad \text{XV.}$$

M is used in this sense in Tables XXI to XXIII.

The average V obtained by this method, at frequencies of 38000~/sec., 29000~/sec., 20700~/sec. was 5.35×10^5 cms./sec. Quoted values ranged from 5×10^5 cms./sec. to 6×10^5 cms./sec. ~~However~~ But, on using a higher frequency of 45400~/sec. ~~the theory broke down~~ discrepancies occurred. V in this case turned out to be 5.12×10^5 cms./sec. Moreover, on repeating the

TABLE XXI

Rod No.	Thick-ness	Width	Freq. Cyc/sec.	λ	Transverse Velocity in Cms/sec x 10^5	M	$\sqrt{E/\rho}$ in cms/sec x 10^5	a/λ
1	0.303 cms.	2.0 cms.	12400	4.9	.605	.111	5.46	.062
			20700	3.75	.775	.145	5.34	.081
			29000	3.16	.917	.171	5.36	.096
			38000	2.74	1.05	.196	5.35	.111
			45400	2.47	1.120	.217	5.12	.123
			62000	2.10	1.30	.253	5.14	.144
			68500	1.99	1.35	.266	5.07	.152
2	0.301	1.4	12400	4.90	.605	.110	5.50	.0614
			20900	3.70	.776	.145	5.35	.0814
			29000	3.14	.910	.1715	5.35	.0960
			37500	2.74	1.03	.195	5.28	.110
			45200	2.47	1.12	.216	5.18	.122
			53700	2.25	1.21	.236	5.12	.134
			69100	1.97	1.36	.267	5.10	.153
			84000	1.76	1.49	.290	5.03	.171
3	0.311	1.0	12650	4.95	.626	.113	5.54	.063
			20700	3.80	.786	.147	5.35	.082
			29000	3.18	.923	.175	5.28	.098
			37500	2.79	1.05	.198	5.28	.107
			45200	2.49	1.13	.222	5.08	.125
			53300	2.28	1.22	.240	5.08	.136
			61500	2.12	1.30	.275	4.95	.157

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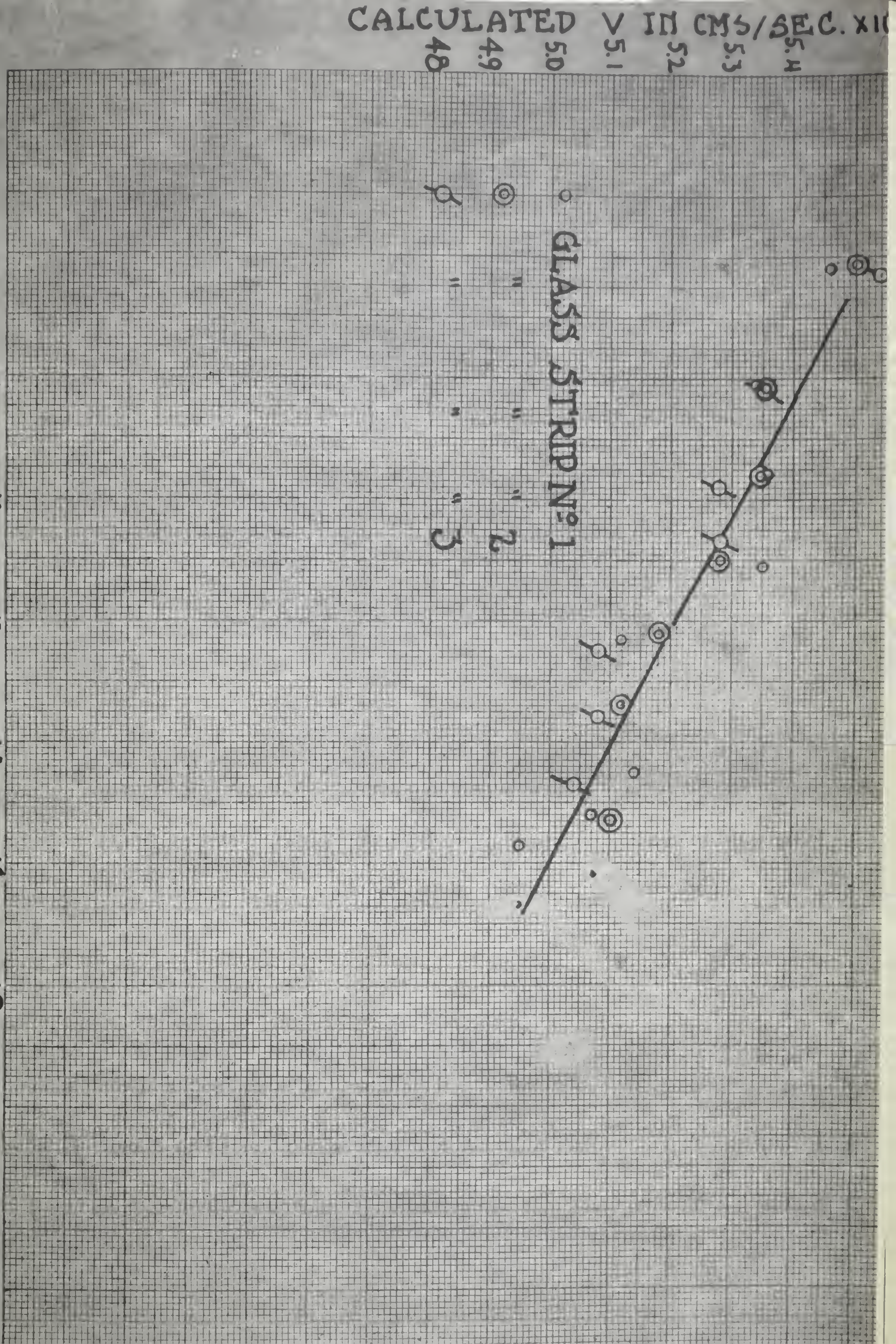
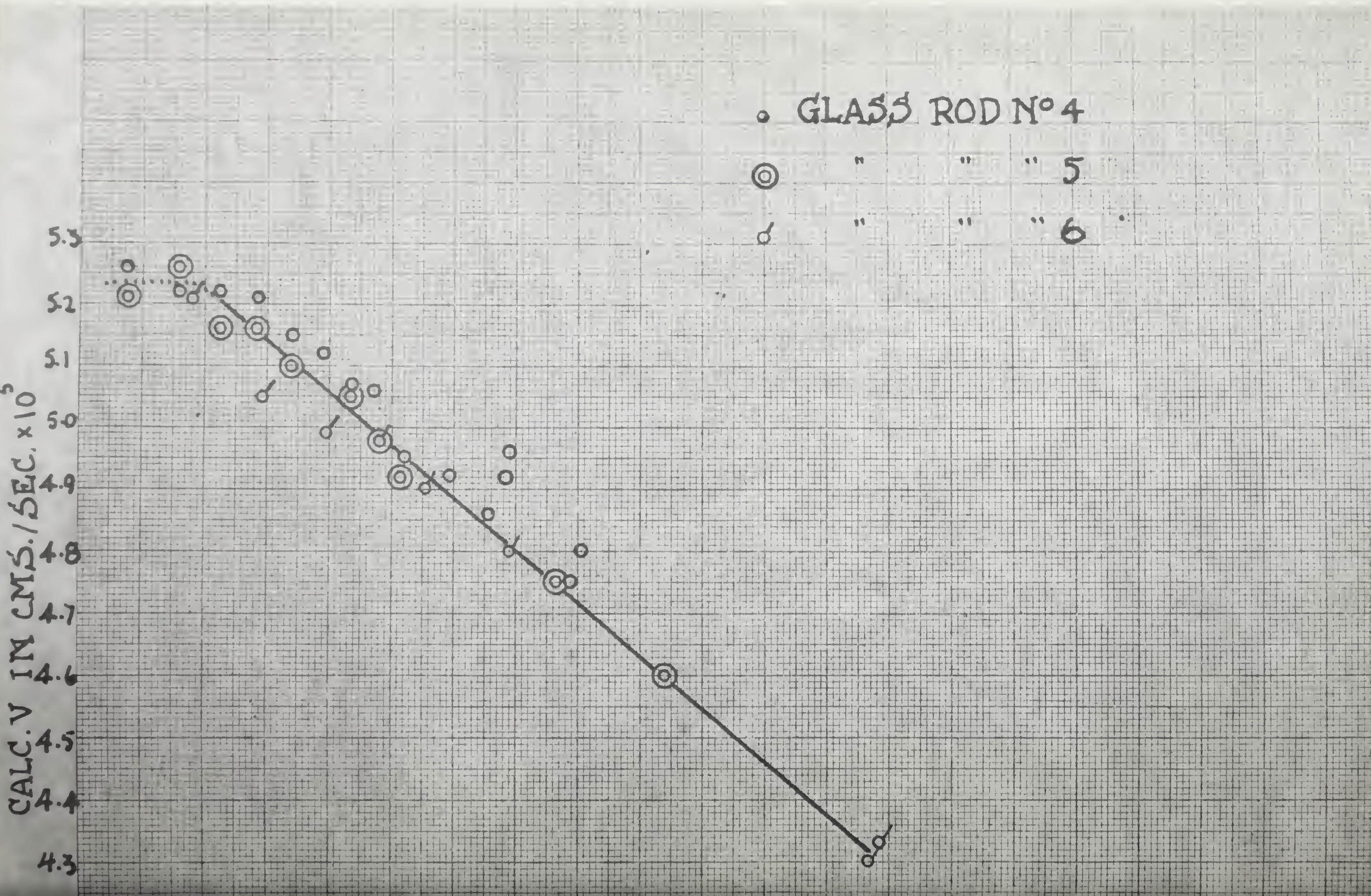


CALCULATED V IN CMS/SEC. X10
4.8
4.9
5.0
5.1
5.2
5.3
5.4
GLASS STRIP Nº 1
" " " 2
" " " 3
.06
.08
.10
.12
.14
.16
.18
RATIO d/λ



GLASS ROD No 4
" " " 5
" " " 6
CALC. V IN CMS./SEC. ×10^5
5.3
5.2
5.1
5.0
4.9
4.8
4.7
4.6
4.5
4.4
4.3

determinations at the former frequencies more carefully than the first time, the good agreement first obtained was found to be accidental, and that in reality V decreased steadily as the frequency was increased.

Now $V = \sqrt{\frac{E}{\rho}}$ must be a constant for a given material, so this decrease in its calculated value should not be taken as an indication that $\sqrt{\frac{E}{\rho}}$ is a function of the frequency but rather that the theory upon which the calculation is based is in error.

According to XIV the velocity of a flexural vibration is not a function of the width of the bar. In order to investigate this point three strips of glass 2 cms. wide, 1.4 cms. wide and 1 cm. wide were cut from the same pane of glass. The pane was so irregular that the thickness varied from strip to strip, as shown in Table XXI. The length of each strip was 91.6 cms. A series of determinations for C and V was made. As is shown in Table XXI and Fig. XXI, these are both independent of the width of the strip. In all three cases, however, V showed the same decrease with increasing frequency. To make the results for different rods comparable, V has been plotted against the ratio of thickness to wave length, a/λ. If the calculated values for V were constant the points would lie on a horizontal line.

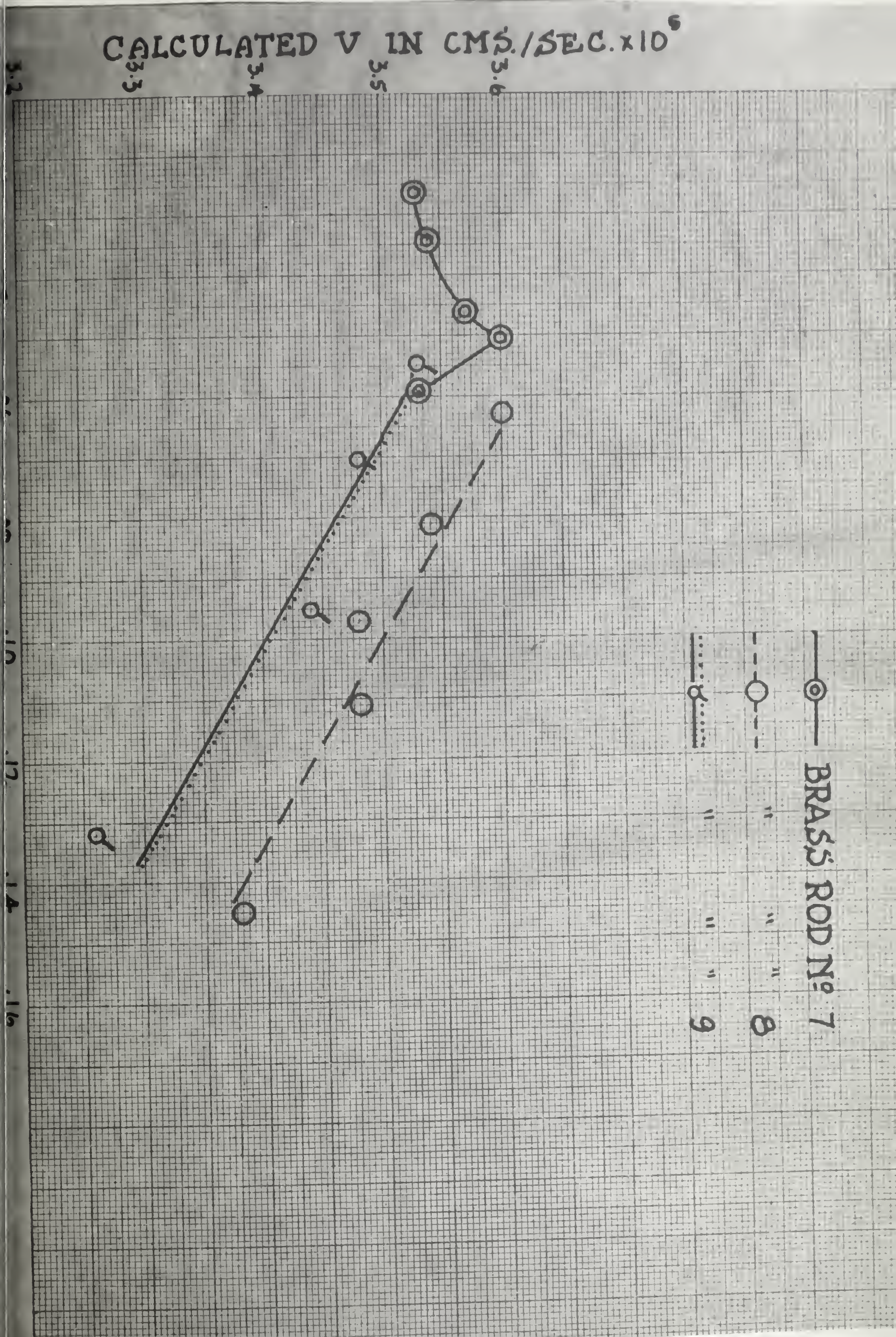
In order to determine the full extent of the departure from theory, strips of thicker and more uniform glass were obtained. The results for these rods are shown in Table XXII and Fig. XXII. It should be possible to extend these curves to the left by using the lower frequencies or thinner rods, or both. At present, facilities for measuring lower frequencies are lacking, and thinner rods of glass are not obtainable. It should also be possible to extend the curves to the right by the use of thicker rods or higher frequencies ~~of~~ or both. However, in this direction, a difficulty has been met with which seems

TABLE XXII

Rod No.	Thick-ness	Width	Freq. Cyc/sec.	λ	Transverse Velocity in Cms./sec. x 10^5	M	$\sqrt{\frac{E}{\rho}}$ in cms/sec x 10^5	a/λ
4	0.255	~~2.35~~	12500	4.23 cms.	.530	.1008	5.26	.055
			20800	3.26	.680	.130	5.22	.072
			29000	2.76	.800	.153	5.22	.085
			37800	2.42	.908	.174	5.21	.097
			45500	2.18	.990	.192	5.15	.108
			53600	1.99	1.07	.209	5.12	.118
			61500	1.85	1.14	.225	5.07	.127
			68500	1.75	1.20	.237	5.06	.134
			77000	1.63	1.25	.252	4.95	.144
			922000	1.48	1.36	.278	4.92	.159
			106000	1.37	1.45	.398	4.86	.171
			113000	1.33	1.50	.305	4.92	.177
			116000	1.32	1.53	.308	4.96	.178
			135000	1.19	1.61	.338	4.75	.198
			141000	1.17	1.65	.342	4.80	.201

TABLE XXII (cont'd)

Rod No.	Thickness	Width	Freq. Cyc/sec.	λ	Transverse Velocity in cms/sec x 10^5	M	$\sqrt{c/\rho}$ in cms/sec x 10^5	a/l
5	0.234	~~2.34~~	12500	4.21 cms.	.525	.1007	5.21	.0555
			20860	3.24	.675	.130	5.26	.0720
			29000	2.73	.790	.153	5.16	.0855
			37500	2.40	.897	.174	5.16	.0970
			45500	2.16	.982	.192	5.10	.1080
			61700	1.84	1.135	.225	5.05	.1270
			69200	1.72	1.190	.239	4.98	.1360
			76200	1.63	1.240	.252	4.92	.1430
			129300	1.21	1.570	.330	4.75	.1930
			173000	1.02	1.770	.384	4.60	.2280
6	0.428	~~0.428~~	12520	5.65	.708	.136	5.21	.076
			20800	4.32	.900	.178	5.05	.099
			29300	3.60	1.055	.211	4.99	.119
			37500	3.16	1.190	.239	4.98	.135
			45500	2.84	1.290	.264	4.90	.151
			61500	2.40	1.480	.308	4.80	.178
			138500	1.46	2.020	.470	4.30	.293
			143000	1.44	2.060	.475	4.33	.297



CALCULATED V IN CMS./SEC.x10⁵
3.2
3.3
3.4
3.5
3.6
.10
.12
.14
.16
BRASS ROD No 7
" " " 8
" " " 9

TABLE XXIII

Rod No.	Thick-ness	Width	Freq. Cyc/sec.	λ	Transverse Velocity in cms/sec x 10^5	M	$\sqrt{\frac{E}{\rho}}$ in cms/sec x 10^5	a/ℓ
7	0.0365	~~0.0365~~	12500	1.370 cms.	.171	.0482	3.53	.0266
			20900	1.060	.221	.0623	3.54	.0344
			37700	.791	.299	.0835	3.57	.0461
			45500	.722	.329	.0913	3.60	.0505
			61500	.615	.378	.1070	3.53	.0593
8	0.203	~~0.203~~	12900	3.20	.412	.114	3.60	.0634
			20550	2.50	.516	.146	3.54	.0812
			29000	2.09	.606	.174	3.48	.0970
			37500	1.83	.686	.197	3.48	.1110
			61500	1.40	.860	.254	3.38	.1450
9	0.149	~~0.149~~	12650	2.74	.347	.0984	3.53	.0545
			20800	2.12	.442	.1270	3.48	.0705
			37200	1.57	.585	.1700	3.44	.0952
			69500	1.13	.782	.2400	3.26	.1320

to be inherent in the behaviour of the wave. To date, it has not been possible to ~~obtain~~ measure the distance between nodes when the frequency has been raised to a point such that a ratio of a/λ greater than 0.30 might be expected. The sand is agitated, but does not become arranged in any regular fashion. The most obvious, but not necessarily correct, conjecture is that the wave does not preserve its form long enough to permit the formation of standing waves. A special research will be necessary in order to clear this point of all undertainty.

A similar series of determinations of the velocity of flexural vibrations in rods of brass was taken, and calculations were made for V, the velocity of a longitudinal vibration in that material. These results are shown in Table XXIII and Fig. XXII.

The results for the thinnest rod (No. 7) all are within 1 % of their mean value, which may be an indication that XIV is a sufficiently close approximation to fact for values of a/λ less than 0.06. It should be noticed that no results for glass have been obtained for values of a/λ less than 0.06. For the other rods, a/λ is greater than 0.06 and the variation in V with increasing values of a/λ is roughly similar to that found for glass. The points of the curves for rod No. 8 and rod No. 9 do not lie on the same line, but this is not surprising as two different samples of rolled brass are almost certain to have slightly different elastic properties produced in them by the process of rolling, while the proportion of metals in the alloy may also differ in the two cases.

The method of rendering the nodes of displacement visible in the experiments on rods suggested a second series of experiments, which constitute one of the easiest methods yet devised for demonstrating <u>visually</u> the properties of wave motion.

A sheet of stiff drawing paper was sealed at one point

FIG. XXIII-A

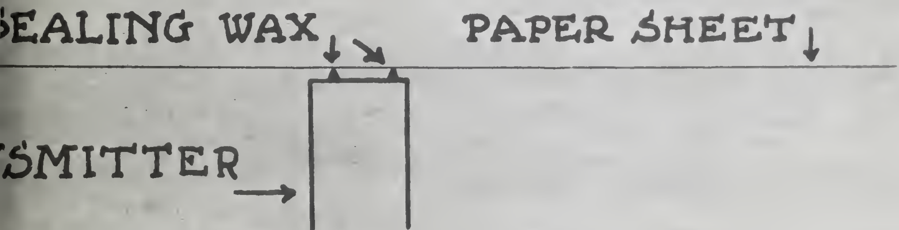


FIG. XXIII-B

FIG. XXIV-A

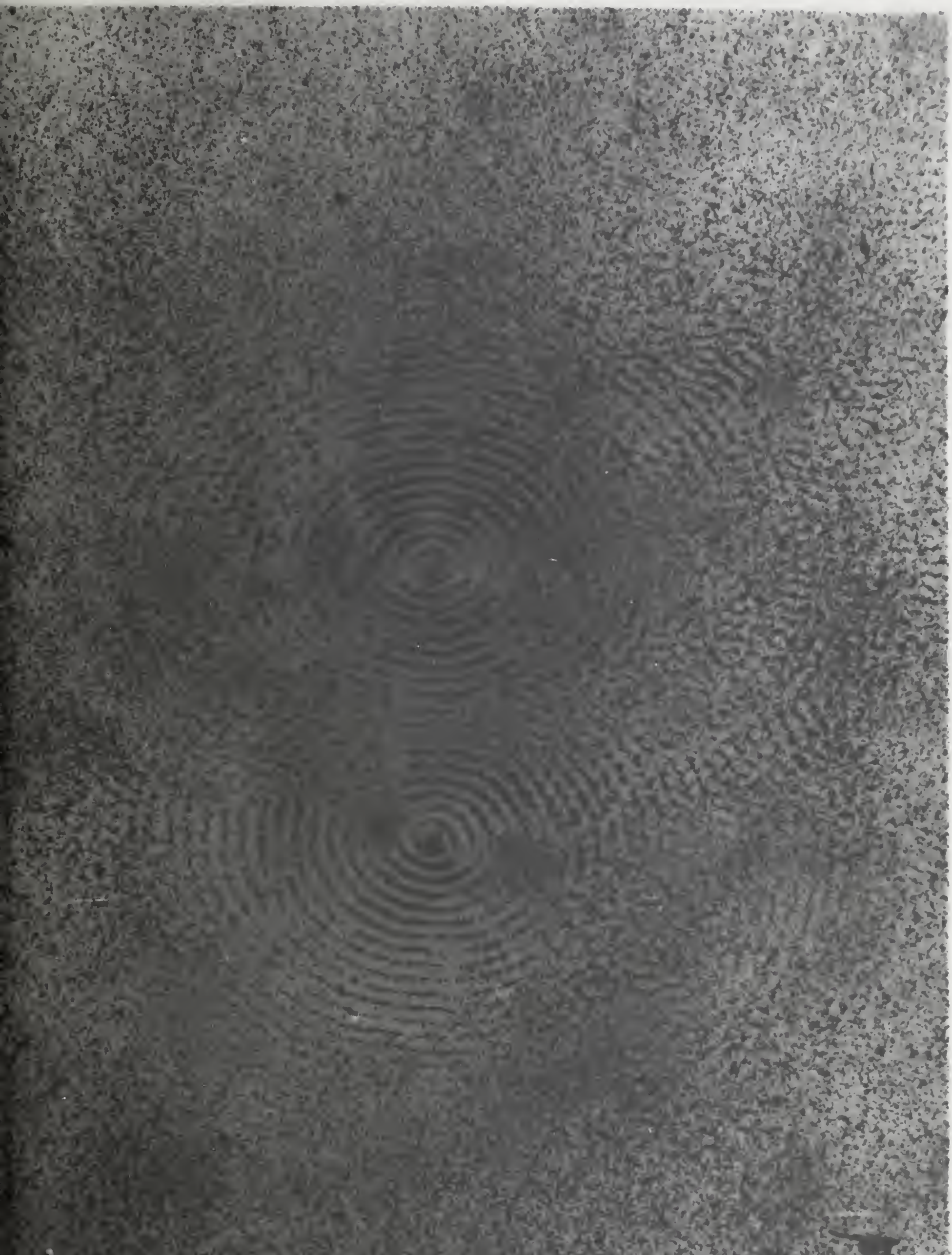
to the face of the transmitter as shown in Fig. XXIII-A, so that the longitudinal vibrations in the rod set up flexural vibrations in the paper. This constitutes a point source emitting two dimensional waves. Sand was placed on the surface of the paper and the point was set vibrating at a frequency of 50000~/sec. The sand became arranged as shown in the photograph in Fig. XXIV-A. There are several points of interest in this arrangement.

1. The standing waves produced apparently are not the result of reflection from the edges of the paper, as the figure remains as shown in the photograph regardless of the manner in which the edge is cut.

2. But, assuming that the distance between nodes is half a wavelength, we have here the evidence of a clear case of double refraction of flexural waves, since the sand figures are not circles but ellipses. This is not surprising as the paper showed a very pronounced "grain" running in the direction of the long axis of the sand figure. The ratio of the fast to slow velocity is approximately 10 : 8.

3. The wave energy does not seem to be distributed uniformly in all directions, as the vibration in the fast velocity direction was not sufficient to move the sand, as is shown by the absence of definite arrangement of the particles on two sides of the vibrating point source. This may be explained if it is assumed that double refraction is really occurring, by pointing out that the energy would naturally be refracted <u>from</u> the axis along which the velocity is a minimum.

A paper specified as ledger index 110, was then sealed to the transmitter in two points as shown in Fig. XXIII-B. The photograph of the sand pattern obtained in this case is shown in Fig. XXIV-B. This shows a series of ellipses with the ratio of major to minor axes



XXIV-B

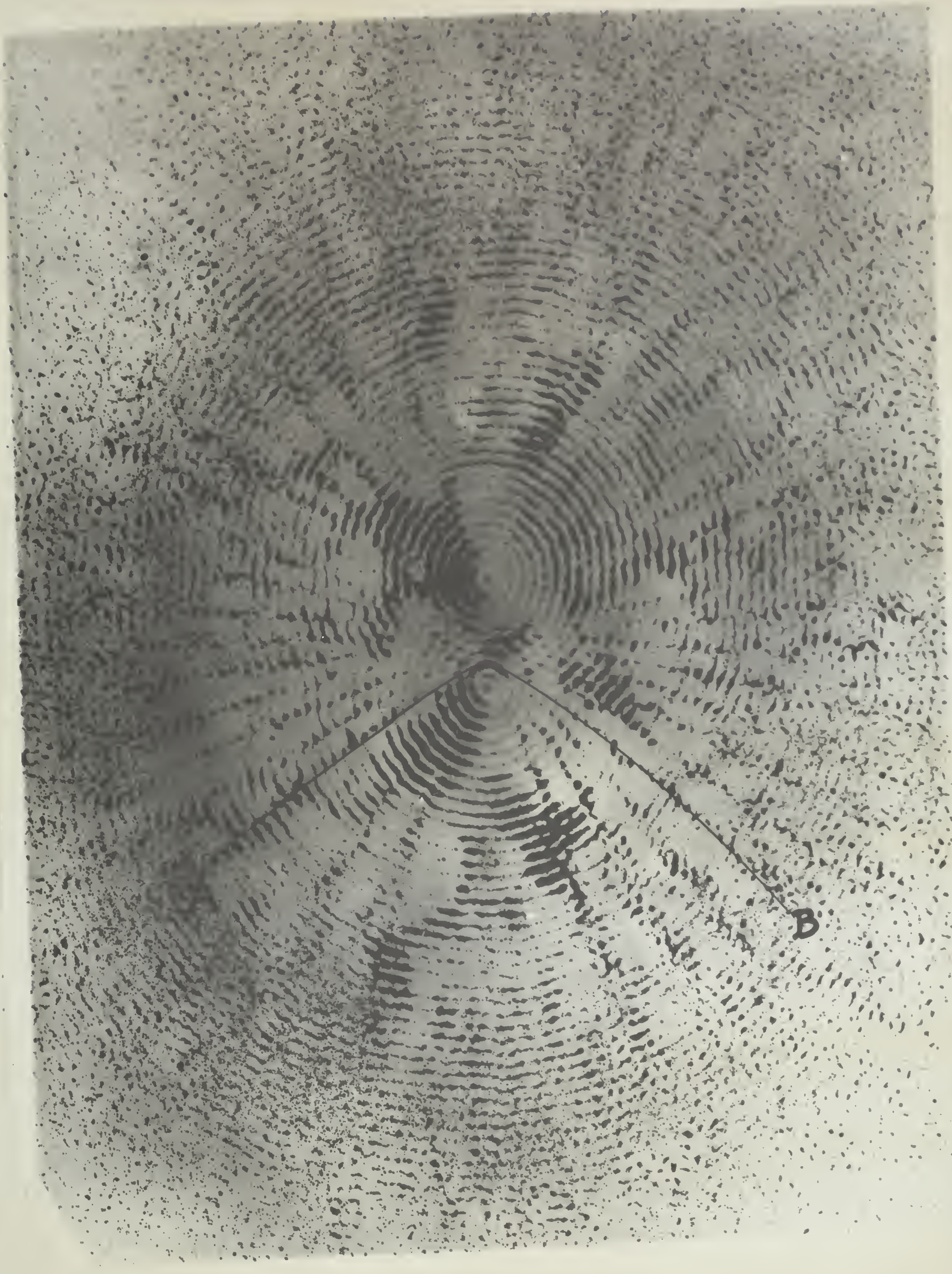


FIG. XXIV-C

about the same as was obtained for the single point source. Faint traces of interference may be seen, but this is much better shown in Fig. XXIV-C. This sand figure was obtained by sealing a sheet of glazed paper 19" x 25" to the transmitter in two points as before. The pattern due to interference is of the type indicated by the curve AB. The same sort of standing wave pattern is formed about each point as in the previous case, but here the ellipses are very nearly, but not quite, concentric circles. The "grain" of this paper was imperceptible, and so double refraction was not expected.

In order to obtain a two dimensional representation of an ultrasonic beam, a sheet of Ledger Index 110 was sealed at one edge to the transmitter and the resulting sand figure obtained asshown in Fig. XXIV-D. Due to damping, only that part of the beam near the transmitter was sufficiently strong to move the sand. However, several points of interest appear.

1. The standing wave system shown is not due to reflection from the edge of the paper opposite the transmitter, as the cutting and tearing of that edge in various ways had no effect on the sand figure shown.

2. Two distinct standing wave systems are produced. One, inside the lines AB and CD (Fig. XXIV-D), and the other outside the lines and differing in phase with the first by a quarter of a wave length.

For comparison, a photo of the visualization of an ultrasonic beam in water is shown in Fig. XXIV-E. This photo was obtained by a method described by Boyle, Lehmann and Reid.[2]

In summary, the work on flexural vibrations has shown:

1. That for waves such that the ratio of thickness to

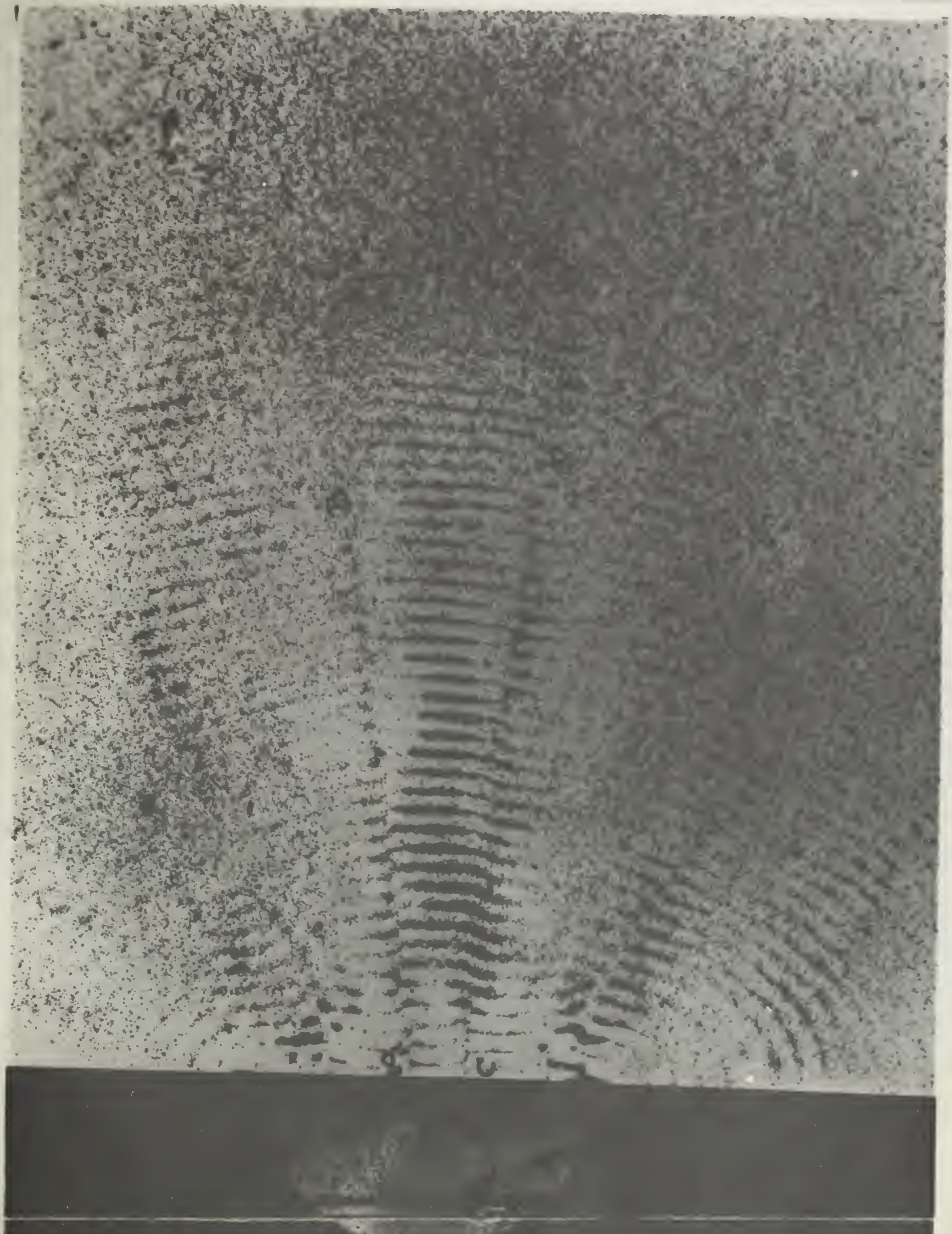


FIG. XXIV-D

FIG. XXIV-E

wave length a/λ is greater than 0.06, the expression given by Lamb for the velocity of a transverse wave requires correction. It has been suggested by Lamb, Love and others, that a correction for rotatory inertia would be required for large values of a/λ, but no indication has previously been given as to how large the ratio would be before the correction would be required.

2. It is probable that theory is in agreement with experiment ~~fact~~ for waves such that a/λ is less than 0.06. Further investigation of this point is necessary.

3. It is probable that harmonic waves cannot be propagated without change of type in a rod for which ~~such that~~ a/λ is greater than 0.3.

4. Experiments on the visualization of wave phenomena making use of the transverse vibration here described may readily be conducted at ultrasonic frequencies using materials that are readily available.

5. New fields of investigation have been opened up, e.g. the correction of classical theory on the propagation of flectural vibrations in bars, the formation of a flectural standing wave system without any apparent reflector, and double refraction of flectural vibrations in non-homogeneous materials.

In conclusion, the writer wishes to express his gratitude to the National Research Council for the award of a Bursary which has made it possible for this work to be carried out, and to thank Dr. R. W. Boyle for suggesting the problem dealt with in Parts I and II of this paper and for the valuable advice and thoughtful direction which he has so freely given throughout the entire investigation.

APPENDIX

Two types of transmitter were employed in Parts I and II of this work and a third type in Part III. Several points arising out of their behaviour merit brief mention here.

One of the types used in Part I possessed no new features. It consisted essentially of two steel plates 1.065 cms. thick and 28.5 cms. in diameter with a mosaic of quartz between them. The whole was sealed as usual in a metal container so that one face only was exposed. The manner of sealing and the precautions to be observed in making transmitters for different purposes has been dealt with by Boyle[1]. The fundamental frequency was about 74,000 cycles ~/sec. The next resonant frequency was about 299,000 cycles~/sec. while the next was at 550,000 cycles ~/sec. This departure from the harmonic relation between succeeding modes of vibration has frequently been noted in the case of double plate transmitters. However, the approximation to the harmonic relation is sufficiently close to warrant the above resonant frequencies. That is, of all the natural modes of vibration of the plate, those excited are the ones for which the quartz is at a node of displacement.

The second type of transmitter involved three steel plates and two quartz mosaics. One of the steel plates was 2.12 cms. thick the other two were the ones employed before, 1.065 cms. thick, i.e. half the thickness of the centre plate. These plates were arranged as shown in Fig. XXV-A.

The faces of each quartz mosaic are differentiated by the fact that a given compression produces a positive charge on one face and a negative charge on the other. The disposition of the positive and negative faces with respect to the steel plates is immaterial in the case of the double plate transmitter, but in the case here considered it becomes

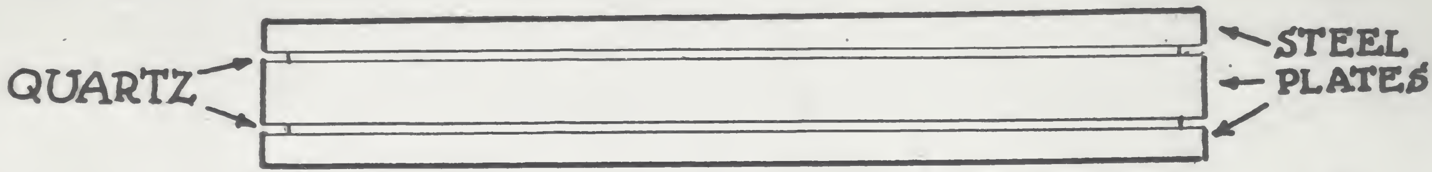


FIG. XXV-A

a factor of considerable importance, as the phase relationship of the disturbance from each plate of quartz depends on the relative position of the positive and negative faces. The manner of connecting the steel plates to the oscillating circuit also alters the phase relationship so that with a given set of quartz and steel plates the following four arrangements are possible.

I. Positive quartz faces toward the centre. The two outside steel plates connected to one side of the oscillating circuit and the centre steel plate to the other terminal.

II. Positive quartz faces toward the centre. Oscillating circuit to the two outside steel plates.

III. One negative and one positive face of quartz plate toward centre. The two outside steel plates to one terminal and the centre steel plate to other.

IV. Quartz faces as in III. One terminal of oscillator to each of the two outside steel plates..

These four cases will be referred to as "arrangement I" - "arrangement II", etc.

Experiment showed that with arrangement I the total thickness is $\lambda/2$, $3/2\lambda$ for the first two modes of vibration

Arrangement II indicated that the plates came into resonance at frequencies such that the total thickness corresponded to λ or 3λ .

Arrangement III - same as II.

Arrangement IV - same as I.

Incidentally, an interesting example of interference was noted.

With the transmitter assembled according to arrangement

ment III, a test for resonance was carried out at the frequency (46,000 cycles/sec.) at which the total thickness of the plates was about $\lambda/2$. No sign of resonance could be detected, which was thought to be due to the existence of internal interferences. In order to verify this opinion, one outside steel plate was short circuited to the centre plate, thus rendering one of the quartz plates inactive, and so incapable of causing interference. Vigorous vibration resulted, due to the unimpeded action of the remaining plate.

For the above transmitters the resonant frequencies occur at intervals of over 100,000 ~/sec. This is a decided disadvantage in conducting experiments demanding the use of a range of frequencies at intervals of say 1,000 or 10,000 cycles per sec., as it necessitates the construction of a large number of different transmitters.

It was possible to overcome this difficulty for the experiments in part III by the use of a transmitter composed of two duralumin rods, each 30.5 cms. long and 1.9 cms. in diameter, sealed with sealing wax to each side of a quartz plate 0.2 cms. thick. The fundamental frequency of such an arrangement is about 4,000 ~/sec. and the succeeding resonant frequencies occur at intervals of about twice the fundamental.

This interval becomes less for succeeding modes of vibration, as has been shown by Boyle and Sproule[15] in an investigation of the effect of later inertia. By the use of rods twice as long the interval between successive resonant frequencies could be reduced to 4,000 ~/sec. The use of rods much longer than this is probably not practicable, due to the effect of damping and the transfer of the energy of the longitudinal vibration into flexural vibration.

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